Quiz of Ordinary Differential Equation II

2018.5.31

1. Consider the equation
$$2x(1+x)y'' + (3+x)y' - xy = 0 \tag{1}$$
   (a) Find the singular points of (1), and verify whether or not these singular points are regular singular points.
   (b) What is the indicial equation of (1) ?
   (c) Write the solutions of (1) in series form.

2. Consider the vectors
$$x^{(1)}(t) = \begin{pmatrix} t^2 \\ 2t \end{pmatrix} \quad \textit{and} \quad x^{(2)}(t) = \begin{pmatrix} e^t \\ e^t \end{pmatrix}$$
   (a) Compute the Wronskian of $x^{(1)}$ and $x^{(2)}$
   (b) In what intervals are $x^{(1)}$ and $x^{(2)}$ linearly independent?
   (c) What conclusion can be drawn about the coefficients in the system of homogeneous differential equation satisfied by $x^{(1)}$ and $x^{(2)}$?
   (d) Find this system of equations and verify the conclusions of part (c).

3. Consider the system
$$x' = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} x \tag{2}$$
   Plot a direction field and determine the qualitative behavior of solutions. Then find the general solution and draw a phase portrait showing several trajectories.

4. Find the general solution of the given system of equations
$$x' = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -8 & -5 & -3 \end{pmatrix} x \tag{3}$$

5. Express the general solution of the given system of equations in terms of real-valued functions.
$$x' = \begin{pmatrix} -3 & 0 & 2 \\ 1 & -1 & 0 \\ -2 & -1 & 0 \end{pmatrix} x \tag{4}$$

6. For the system

$$x' = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} x \tag{5}$$

   (a) Find a fundamental matrix and the general solutions for the system (8)
   (b) Find the fundamental matrix Φ such that $\Phi(0) = I$
   (c) Find e^A ?

7. Find a fundamental set of solutions of

$$x' = Ax = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} x \tag{6}$$

   and draw a phase portrait for this system.

8. Find the solution of the following initial value problem:

$$x' = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 3 & 6 & 2 \end{pmatrix} x, \qquad x(0) = \begin{pmatrix} -1 \\ 2 \\ -30 \end{pmatrix} \tag{7}$$

9. Use the method of undetermined coefficients to find a particular solution of

$$x' = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} x + \begin{pmatrix} 2e^{-t} \\ 3t \end{pmatrix} = Ax + g(t) \tag{8}$$

10. Verify that the given vectors is the general solution of the corresponding homogeneous system, and then solve the nonhomogeneous system. Assume that $t > 0$

$$tx' = \begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix} x + \begin{pmatrix} 1 - t^2 \\ 2t \end{pmatrix} \tag{9}$$

$$x^{(c)} = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} t + c_2 \begin{pmatrix} 1 \\ 3 \end{pmatrix} t^{-1} \tag{10}$$