7選5 作答

Mid-Exam of Ordinary Differential Equations (P1)

(Class A, Nov. 15, 2017)

1. (Construct and Investigate the model of Mixing)

At time $t=0$ a tank contains Q_0 lb of salt dissolved in 100 gal of walter.

Assume that walter containing $\frac{1}{4}$ lb of salt per gallon is entering the tank at a rate of r gal/min and that the well-stirred mixture is draining from the tank at the same rate.

(a) Set up the initial value problem that describe this flow process.

(b) Find $Q(t)$: the amount of salt in the tank at time t.

(c) Find $\lim_{t\to\infty} Q(t)$.

2. Solve the following initival value problems respectively.

(1) $\begin{cases}(16-t^2)y' + 2ty = 3t^2 \\ y(1) = -5\end{cases}$

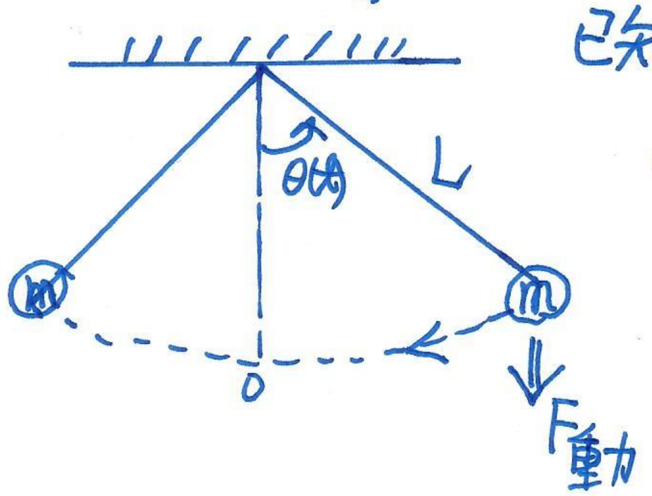
(2) $\begin{cases}2y' + ty = 2 \\ y(0) = 1\end{cases}$

(3) $\dfrac{dy}{dx} = \dfrac{4x - x^3}{4 + y^3}$, $y(0) = 1$

(4) $\begin{cases}(3xy + y^2) + (x^2 + xy)\, y' = 0 \\ y(1) = 1\end{cases}$

3. 探討下列問題的微分方程 models

(A) 同一單擺分割置於地球表面及月球表面作來回運動模型.(不計摩擦力及地球上的空氣阻力) 當此單擺是作微小角度擺動時,試比較其週期性.(已知月球質量大約$=\frac{1}{81}$地球質量)



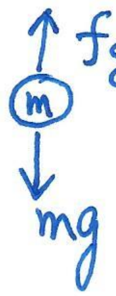
已知:

$|\theta(t)| < \theta_0 << 1 \quad \forall t$

$F_{(\text{重力})_{\text{地}}} = m g_{\text{地球}}$

$F_{(\text{重力})_{\text{月}}} = m g_{\text{月球}}$

(B) A Falling Object:



Assume

$f_d \propto v$ (Case 1)

$f_d \propto v^2$ (Case 2)

4. Solve the following ODE respectively. Find $\lim_{t\to\infty} y(t)$.

(1) $\frac{dy}{dt} = -r\left(1-\frac{y}{T}\right)y$, $y(0)=y_0>0$, r, T: positive constant.

(2) $\frac{dy}{dt} = -r\left(\frac{y}{T}-1\right)\left(\frac{y}{K}-1\right)y$, $r>0$, $0<T<K$: constants

5. Solve the following type of Bernoulli's equation

$$\begin{cases} t^2 y' + 2ty - y^3 = 0, \ t > 0 \\ y(0) = y_0 \end{cases}$$

Hint: Use the substitution $v = y^{-2}$.

6. Solve the initial value problem

$$\begin{cases} y' = \frac{-y}{3} + t \equiv f(t, y) \\ y(0) = 0 \end{cases}$$

by the method of following successive approximation:

$$\phi_0(t) = 0$$

$$\phi_1(t) = \int_0^t f(s, \phi_0(s))\, ds$$

$$\phi_2(t) = \int_0^t f(s, \phi_1(s))\, ds$$

$$\vdots$$

$$\phi_{k+1}(t) = \int_0^t f(s, \phi_k(s))\, ds, \quad k = 0, 1, 2, \cdots.$$

Find $\lim_{k \to \infty} \phi_k(t)$

7. Consider the (IVP) $\begin{cases} \frac{dy}{dt} = 3 - 2t - 0.5y \\ y(0) = 1. \end{cases}$

Use Euler's method: $y_{n+1} = y_n + f(t_n, y_n)(t_{n+1} - t_n)$, $n = 0, 1, 2 \cdots$;
with step size $h = 0.2$ to find approximation of the solution at $t = 0.2, 0.4, 0.6, 0.8$.