(7選5題作答)

Class

Midterm1 of ordinary differential equation (B)

November ~~14~~ 15, 2017

1. Consider the equation

$$\frac{dy}{dx} = \frac{y-4x}{x-y} \tag{1}$$

   (a) Show that equation (1) can be rewritten as

$$\frac{dy}{dx} = \frac{(y/x)-4}{1-(y/x)} \tag{2}$$

   (b) Introduce a new dependent variable v so that $v = y/x$, or $y = xv(x)$. Expressed dy/dx in terms of x, v, and dv/dx

   (c) Replace y and dy/dx in equation (2) by the expressions from partb that involve v and dv/dx. Show that the resulting differential equation is

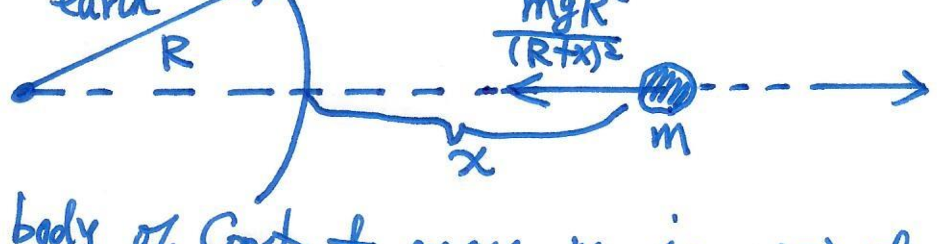
$$v + x\frac{dv}{dx} = \frac{v-4}{1-v}$$

2. Consider the differential equation $dy/dt = ay - b$

   (a) Find the equilibrium solution y_e

   (b) Let $Y(t) = y - y_e$; thus $Y(t)$ is the deviation from the equilibrium solution. Find the differential equation satisfied by $Y(t)$.

3. A body in the earth's gravitational field is pulled toward's the center of the earth.



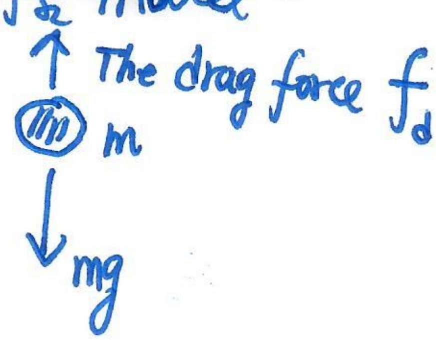
A body of constant mass m is projected from the earth in a direction perpendicular to the earth's surface with an initial velocity V_0. Assuming that there is no air resistance

Prove: (1) The maximum altitude $A_{max} = \frac{V_0^2 R}{2gR - V_0^2}$

(2) The escape velocity $v_e = \sqrt{2gR}$.

4. 探究下列問題的微分方程 model：

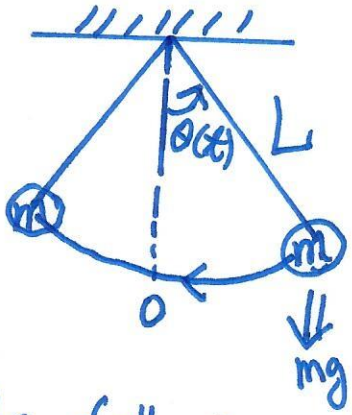
(A) A Falling Object.



Assume： $f_d \propto v$ (Case 1)

$f_d \propto v^2$ (Case 2)

(B) 地球表面上，單擺來回運動模型.



5 Solve the following Differential Equations：

(1) Find the general solution of

$$\frac{dy}{dx} + \frac{1}{2}y = \frac{1}{2}e^{x/3}$$

(2) $$\frac{dy}{dx} = \frac{3x^2+4x+2}{2(y-1)}, \quad y(0) = -1$$

(3) $$(y\cos x + 2xe^y) + (\sin x + x^2e^y - 1)\,y' = 0$$

6. Solve the following equations respectively：

Find $\lim_{t\to\infty} y(t)$

(1) $$\frac{dy}{dt} = r\left(1 - \frac{y}{K}\right)y, \quad y(0) = y_0 > 0 \quad \text{where } r, K: \text{positive constants}$$

(2) $$\begin{cases} \dfrac{dy}{dt} = -r\left(1 - \dfrac{y}{T}\right)\left(1 - \dfrac{y}{K}\right)y \\ y(0) = y_0 > 0 \end{cases} \quad \text{where } r > 0,\ 0 < T < K \text{ constants}$$

7. Solve the initial value problem

$$\begin{cases} y' = 2t\,(1+y) \equiv f(t,y) \\ y(0) = 0 \end{cases}$$

by the method of following successive approximations:

$$\phi_0(t) = 0$$

$$\phi_1(t) = \int_0^t f(s, \phi_0(s))\,ds$$

$$\phi_2(t) = \int_0^t f(s, \phi_1(s))\,ds$$

$$\vdots$$

$$\phi_{k+1}(t) = \int_0^t f(s, \phi_k(s))\,ds, \quad k = 0, 1, 2, \cdots.$$

Find $\lim\limits_{k\to\infty} \phi_k(t)$.