REAL ANALYSIS MIDTERM II

1. Let D be a measurable subset in R^d, and f, g are measurable functions on D, show that
   (a) $f + cg$ for some constant c
   (b) fg
   (c) $\dfrac{f}{g}$ for $g \neq 0$ almost everywhere.
   (d) $\{f > g\}$
   are measurable

2. (a) Show that the limit of an increasing sequence of functions lsc at x_0 is lsc at x_0
   (b) Let f be lsc and larger than $-\infty$ on $[a, b]$. Show that there exist continuous f_k on $[a, b]$ such that $f_k \nearrow f$.

3. Suppose $f_k \to f$ and $g_k \to g$ in measure on E. Show that
   (a) $f_k + g_k \to f + g$ in measure on E.
   (b) If $|E| < +\infty$, then $f_k g_k \to fg$ in measure on E.

4. True or False:
   (a) Let f and f_k, $k = 1, 2, ...$ be measurable and finite a.e. in E. If $f_k \to f$ a.e. on E, then $f_k \to f$ in measure on E.
   (b) If $f_k \to f$ in measure on E, then $f_k \to f$ a.e. on E.
   If ture, show the proof, if false, give an counterexample.

5. Let f be usc and less than $+\infty$ on a compact set E. Show that there exists $x_0 \in E$ such that $f(x_0) \geq f(x)$ for all $x \in E$.

6. State the following two theorems:
   (a) Egorov's Theorem
   (b) Lusin's Theorem