REAL ANALYSIS FINAL EXAM.

1. If $\{x_n\}$ is a sequence of strictly positive real numbers, then show that
$$liminf \frac{x_{n+1}}{x_n} \leq liminf x_n^{1/n} \leq limsup x_n^{1/n} \leq limsup \frac{x_{n+1}}{x_n}$$
(Show your work by the $\epsilon - N$ definition of limsup and liminf)

2. Let f be measurable and finite a.e. in E and $|E| < +\infty$ If $a < f(x) \leq b$ (a and b are finite) for $x \in E$, then show that
$$\int_E f = -\int_a^b \alpha d\omega(\alpha)$$
where ω is a distribution function of f on E.

3. True or false: Let f be a function whose improper Riemann integral exists and is finite, then f is Lebesgue integrable.
(If ture, show the proof, if false, give a counterexample.)

4. True or False:
   (a) Let f and f_k, $k = 1, 2, ...$ be measurable and finite a.e. in E. If $f_k \to f$ a.e. on E, then $f_k \to f$ in measure on E.
   (b) If $f_k \to f$ in measure on E, then $f_k \to f$ a.e. on E.
   If ture, show the proof, if false, give a counterexample.

5. State the following Lemma and Theorems:
   (a) Fatou's Lemma
   (b) Lebesgue's Dominated Convergence Theorem.
   (c) Monotone Convergence Theorem
   (d) Uniform Convergence Theorem
   (You only need to state, don't prove!)

6. (a) If f,$g \in L(E)$, then show that $f + g \in L(E)$.
   (b) If $f \in L(E)$, g is measruable on E, and there exists a finite constant M such that $|g| \leq M$ a.e. in E, then show that $fg \in L(E)$.

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