Major Research Achievements

一、Nonlinear Hyperbolic Partial Differential Equations:

1. Gas Dynamical Combustion:

We study the Chapman Jouguet (CJ) model and the selfsimilar Zeldovich von Neumann Doring (SZND) model in chemically reacting gas flows. We discover some basic relationships among ignition temperature $T_i$, total chemical binding energy $Q$, and the adiabatic exponent $\gamma$ of polytropic gas. From these relations, we can determine when temperatures along the (SZND) burning solutions are higher than the ignition temperature $T_i$. We also study the all possible selfsimilar solutions for the (SZND) model. From these results, we can determine when selfsimilar solutions for the (CJ) model are the limits of selfsimilar solutions of the (SZND) model when the reaction rate tends to infinity.

2. Relativistic Euler Equations:

This work considers a more realistic equation of states. Some useful entropy-entropy flux and the approximation solutions are constructed by Godunov scheme. Then we establish an integration formula for solutions of the relativistic Euler-Poisson-Darboux equation. By this integral representation, we generalize the method of DiPerna and G.-Q. Chen et al to show the existence of weak solution of the relativistic Euler equation with initial data containing the vacuum state.

3. General Relativity: Tolman-Oppenheimer-Volkoff-de Sitter Equation:

It's known that spherically symmetric static solutions of the Einstein equations with a positive cosmological constant for the energy-momentum tensor of a barotropic perfect fluid are governed by the Tolman-Oppenheimer-Volkoff-de Sitter equation. Some sufficient conditions for the existence of monotone-short solutions of the equation are given in this article. Then we show that the interior metric can extend to the exterior Schwarzschild-de Sitter metric on the exterior vacuum region with twice continuous differentiability. In addition, we investigate the analytic property of the solutions at the vacuum boundary. Our result can be considered as the de Sitter version of the result by A. D. Rendall and G. B. Schmidt (1991). Furthermore, one can see that there are different properties of the solutions with those of the Tolman-Oppenheimer-Volkoff equation (with zero cosmological constant) in certain situation.

4. Viscous Nozzle Flows and Traffic Flows:

(1) We consider the existence and stability for steady states of one-dimensional viscous isentropic compressible flows through a contracting-expanding or expanding-contracting nozzle. Treating the viscosity coefficient as a singular parameter, the steady-state problem can be viewed as a singularly perturbed system. For such nozzles, a complete classification of steady states is given and the existence of viscous profiles is established via the geometric singular perturbation theory. Particularly interesting is the existence of a maximal sub-to-super transonic wave for contracting-expanding nozzle and its role in the formation of other complicated transonic waves consisting of a sub-to-super portion. These sub-to-super transonic steady states are newly founded by the authors using the geometric singular perturbation theory. Applying the energy method and Sobolev imbedding theorem, we show that the sub-to-super inviscid transonic stationary wave is physically relevant in the sense that it is L-infinity linear stable on any bounded space interval. Moreover, generalized inviscid stationary waves are also classified for discontinuous and expanding or contracting nozzles by the limiting argument and Helly's selection principle. A new entropy condition is imposed to select a unique admissible standing shock in generalized stationary wave. We show that, such admissible solution selected by the entropy condition, admits minimal total variation and has minimal enthalpy loss across the standing shock in the limiting process.

(2) We consider the existence and stability of stationary waves for viscous traffic flow models. From the viewpoint of dynamical systems, the steady-state problem of the systems can be formulated as a singularly perturbed problem. Using the geometric singular perturbation method, we establish the existence of stationary waves for both the inviscid and viscous systems. The inviscid stationary waves contain smooth waves and discontinuous transonic waves. Both waves admit viscous profiles for the
viscous systems. Then we consider the linearized eigenvalue problem of the systems along smooth stationary waves. Applying the technique of center manifold reduction, we show that any supersonic smooth stationary wave is spectrally unstable.

5. Quasilinear Wave Equations:

(1) We study one-dimensional motions of polytropic gas governed by the compressible Euler equations. The problem on the half space under a constant gravity gives an equilibrium which has free boundary touching the vacuum and the linearized approximation at this equilibrium gives time periodic solutions. However, it is difficult to justify the existence of long-time true solutions for which this time periodic solution is the first approximation. The situation is in contrast to the problem of free motions without gravity. The reason is that the usual iteration method for quasilinear hyperbolic problem cannot be used because of the loss of regularities which causes from the touch with the vacuum. Due to this reason, we try to find a family of solutions expanded by a small parameter and apply the Nash-Moser Theorem to justify this expansion. Note that the application of Nash-Moser Theorem is necessary for the sake of conquest of the trouble with loss of regularities, and the justification of the applicability requires a very delicate analysis of the problem.

(2) We investigate the existence of globally Lipschitz continuous solutions to a class of Cauchy problem and initial-boundary value problem of quasilinear wave equations. Applying Lax’s method and generalized Glimm’s method, we construct the approximate solutions of the corresponding perturbed Riemann problem for the Cauchy problem, and initial-boundary Riemann problem near the boundary layer. Then we establish the global existence for the derivatives of solutions. The existence of global Lipschitz continuous solutions can be carried out by showing the weak convergence of residuals for the source term of equations.

二、Reaction-Diffusion Equations

1. Waves Propagation of Epidemic Models:

(1) We investigate the existence, uniqueness, monotonicity and asymptotic behaviour of travelling wave solutions for a general epidemic model arising from the spread of an epidemic by oral-faecal transmission. First, we apply Schauder's fixed point theorem combining with a supersolution and subsolution pair to derive the existence of positive monotone monostable travelling wave solutions. Then, applying the Ikehara's theorem, we determine the exponential rates of travelling wave solutions which converge to two different equilibria as the moving coordinate tends to positive infinity and negative infinity, respectively. Finally, using the sliding method, we prove the uniqueness result provided the travelling wave solutions satisfy some boundedness conditions.

(2) We study the existence of entire solutions for delayed monostable epidemic models with and without the quasi-monotone condition. In the quasi-monotone case, we first establish the comparison principle and construct appropriate sub-solutions and upper estimates. Then the existence and qualitative features of entire solutions are proved by mixing any finite number of traveling wave fronts with different speeds $c \geq c^*$ and directions and a spatially independent solution, where $c^* > 0$ is the critical wave speed. In the non-quasi-monotone case, some new types of entire solutions are constructed by using the traveling wave fronts and spatially independent solutions of two auxiliary quasi-monotone systems and a comparison theorem for the Cauchy problems of the three systems.

2. Spatial Dynamics of Delayed Nonlocal Reaction-Diffusion System:

We study the spatial dynamics of some delayed nonlocal reaction-diffusion systems in whole space. We first establish a series of comparison theorems to investigate the global attractivity of the equilibria for a delayed nonlocal reaction-diffusion system with and without quasi-monotonicity. Then we show that the spreading speed of a general system without quasi-monotone conditions is coincident with the minimal wave speed. Applying a fluctuation method, we further provide some sufficient conditions to ensure the upward convergence of the spreading speed and traveling wave solutions. Finally, we point out the effects of the delay and nonlocality on the spreading speed of the non-quasi-monotone systems.

3. FitzHugh–Nagumo Type Equations:

We consider the diversity of traveling wave solutions of the FitzHugh-Nagumo type equations $u_t = u_{xx} +$
\( f(u,w), \ w=eg(u,w), \) where \( f(u,w)=u(u-a(w))(1-u) \) for some smooth function \( a(w) \) and \( g(u,w)=u-w. \) When \( a(w) \) crosses zero and one, the corresponding profile equation possesses special turning points which result in very rich dynamics. By using the geometric singular perturbation theory, we study the co-existence of different traveling waves whose slow orbits could involve all portions of the slow manifold. We give a complete classification of all different fronts of traveling waves, and provide an example to support our theoretical analysis.

4. Cooperative and Competitive Biological Models:

We investigate the existence of traveling wave solutions for a class of diffusive predator-prey type systems whose each nonlinear term can be separated as a product of suitable smooth functions satisfying some monotonic conditions. The profile equations for the above system can be reduced as a four-dimensional ODE system, and the traveling wave solutions which connect two different equilibria or the small amplitude traveling wave train solutions are equivalent to the heteroclinic orbits or small amplitude periodic solutions of the reduced system. Applying the methods of Wazewski Theorem, LaSalle’s Invariance Principle and Hopf bifurcation theory, we obtain the existence results. Our results can apply to various kinds of ecological models.

三、Dynamical Systems and Cellular Neural Networks

1. Wave Propagation of Lattice Dynamical System:

(1) We establish the existence and nonexistence of traveling waves for nonlinear cellular neural networks finite or infinite distributed delays. The dynamics of each given cell depends on itself and its nearest \( m \) left or \( l \) right neighborhood cells, where delays exist in self-feedback and left or right neighborhood interactions. Our approach is to use Schauder's fixed point theorem coupled with upper and lower solutions of the integral equation in a suitable Banach space. Further, we obtain the exponential asymptotic behavior in the negative infinity and the existence of traveling waves for the minimal wave speed by the limiting argument. In addition, we also use the traveling wave solutions to prove the existence of entire solutions. The entire solutions are defined in the whole space and for all time. From our previous work, we know that the DCNN model admits traveling front solutions. Combining the traveling front solutions with different wave speeds and a spatially independent solution of the DCNN model, we establish some new entire solutions to describe the interactions of traveling fronts. Various qualitative features of the entire solutions are also investigated.

(2) We are interested in finding entire solutions of a bistable periodic lattice dynamical system. By constructing appropriate super- and subsolutions of the system, we establish two different types of merging-front entire solutions. The first type can be characterized by two monostable fronts merging and converging to a single bistable front; while the second type is a solution which behaves as a monostable front merging with a bistable front and one chases another from the same side of the lattice. Due to the periodicity of the system, we have to emphasize that there has no symmetry between the increasing and decreasing pulsating traveling fronts, which increases the difficulty of construction of the super- and subsolutions.

(3) We consider the existence of traveling plane wave solutions of a class of delayed lattice differential system in Lotka-Volterra type and Kolmogorov-type. Employing the techniques of cross iteration method coupled with the explicit construction of upper and lower solutions in the theory of weak quasi-monotone dynamical systems, we obtain a critical speed, and show the existence of traveling plane wave solutions connecting two different equilibria when the wave speeds are less than the critical speed.

(4) We establish the existence, uniqueness, asymptotic behavior and stability of traveling waves for delayed cellular neural networks with monotone or non-monotone output functions. For non-monotone output functions, the key techniques are to sandwich the given output function between two appropriate non-decreasing functions and to use Schauder's fixed theorem in a suitable Banach space. Moreover, we use the weighted energy and comparison principle to prove the global stability of traveling wave solutions.

2. Random Attractors and Stability Problems:
We consider the existence, tempered behavior and fractal dimension of pullback global attractors for some partial differential equations, e.g. the non-homogeneous discrete Klein-Gordon-Schrodinger type equations, the non-autonomous micropolar fluid flows and incompressible non-Newtonian fluid with delayed external force.

We investigate the dissipative dynamical system in the infinite lattice $\mathbb{Z}$. The dynamics of each node depends on itself and nearby nodes by a nonlinear function. When each node is perturbed with weighted Gaussian white noise, a unique pullback attractor and forward attractor exists whose domain of attraction are random tempered sets. Furthermore, we prove that the pullback and forward attractors are equivalent to a random equilibrium which is also tempered. Both convergences to the pullback and forward attractors are exponentially fast.

We investigate the global stability of cellular neural networks with distributive time delays and noise perturbations. Two types of noise perturbations are considered in the system, one is internal noise and another is external noise. We show that the deterministic system preserves the globally exponential stability when it is perturbed by the internal noise with small noise strength. On the other hand, the deterministic system can only preserve weaker stability when it is perturbed by the external noise. We also provide some numerical simulations to support our theoretical analysis.

We study the existence, uniqueness and stability of periodic solutions for a two-neuron network system with or without external inputs. The system consists of two identical neurons, each possessing nonlinear feedback and connected to the other neuron via a nonlinear sigmoidal activation function. In the absence of external inputs but with appropriate conditions on the feedback and connection strengths, we prove the existence, uniqueness and stability of periodic solutions by using the Poincaré-Bendixson theorem together with Dulac's criterion. On the other hand, for the system with periodic external inputs, combining the techniques of the Liapunov function with the contraction mapping theorem, we propose some sufficient conditions for establishing the existence, uniqueness and exponential stability of the periodic solutions. Some numerical results are also provided to demonstrate the theoretical analysis.

3. Spatial Chaos of Cellular Neural Networks:

(1) We studied the spatial disorder of Cellular Neural Networks by reducing the problem to one or two dimensional maps. Applying the kneading theory and iteration map method, the spatial entropy of one-dimensional map can be obtained explicitly as a staircase function. For two-dimensional map, we derived the Smale horseshoe structure. There results are closely related to Henón-type map, Belykh map, and discrete Allen-Cahn equations. In addition, we generalized these results to a certain class of gap maps and demonstrated the devil’s staircase structure of topological entropy function. These results are helpful in applications of communication of chaos.

(2) We investigate the complexity of one-dimensional cellular neural network mosaic patterns with spatially variant templates on finite and infinite lattices. Various boundary conditions are considered for finite lattices and the exact number of mosaic patterns is computed precisely. The entropy of mosaic patterns with periodic templates can also be calculated for infinite lattices. Furthermore, we show the abundance of mosaic patterns with respect to template periods and, which differ greatly from cases with spatially invariant templates.