vector space

Def: Let V be a VS over F and $S \subseteq V$.

S is said to be linearly dependent (l.d. for short)

if $\exists$ <u>distinct</u> vectors $\alpha_1, \alpha_2, \cdots, \alpha_n \in S$

and $\exists$ scalars $a_1, \cdots, a_n \in F$, not all zero,

s.t. $$a_1\alpha_1 + a_2\alpha_2 + \cdots + a_n\alpha_n = 0$$

A set which is not l.d. is called

linearly independent. (l.i. for short)

Remark: If $S = \{\alpha_1, \alpha_2, \cdots, \alpha_l\}$ we

sometimes say that $\alpha_1, \alpha_2 \ldots \alpha_l$ are

l.d. (or l.i.).

Exa2: let $A = \begin{pmatrix} 1 & -3 & 2 \\ -4 & 0 & 5 \end{pmatrix}$

$B = \begin{pmatrix} -3 & 7 & 4 \\ 6 & -2 & -7 \end{pmatrix}$

$C = \begin{pmatrix} -2 & 3 & 11 \\ -1 & -3 & 2 \end{pmatrix}$ be vectors in $M_{2\times3}(\mathbb{R})$.

Then A, B, C are l.d. since

$$5A + 3B - 2C = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$ zero vector in $M_{2\times3}(\mathbb{R})$

Fact

(1) Any set containing the 0 vector is l.d.

(2) The empty set is l.i.

(3) If u is not a zero vector then $\{u\}$ is l.i

Thm 1.6 (4)[P39] Any set which contins a l.d. set is l.d.

(5)[P39] Any subset of a l.i. set is l.i.

(6) A set S of vectors is l.i. $\Leftrightarrow$ for any distinct vectors $\alpha_1, \alpha_2, \cdots, \alpha_n$ of S,

$c_1\alpha_1 + c_2\alpha_2 + \cdots + c_n\alpha_n = 0$ implies

each $c_i = 0$.

Exa3[P38] Let $S = \left\{ \underset{\alpha}{\begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix}}, \underset{\beta}{\begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix}}, \underset{\gamma}{\begin{pmatrix} 0 \\ 0 \\ 1 \\ -1 \end{pmatrix}}, \underset{\delta}{\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}} \right\}$.

Then S is l.i. Indeed,

since $a_1\alpha + a_2\beta + a_3\gamma + a_4\delta = 0$

implies $a_1 = a_2 = a_3 = a_4 = 0$

Fact (7) vectors $\alpha_1, \alpha_2, \cdots, \alpha_n$ are l.i.

$\Leftrightarrow$ $c_1\alpha_1 + c_2\alpha_2 + \cdots + c_n\alpha_n = 0$ implies

$c_1 = c_2 = \cdots = c_n = 0$

Thm 1.7 p39 Let S be a l.i. subset of V.

Let $v \in V - S$. Then

$S \cup \{v\}$ is l.d. $\Leftrightarrow$ $v \in \text{span}(S)$

Remark：課本的証明有缺失！

Remark：一般是反述來用這 Thm.

Fact: If no proper subset of S generates the span of S, then S must be l.i. .