Exercise (A)

Problem 1: Prove that $m(3)=7$.

Problem 2: Prove that

1. $\left(\frac{n}{e}\right)^n < n! < en\left(\frac{n}{e}\right)^n$, $n>1$
2. $\left(\frac{n}{k}\right)^k < \binom{n}{k} < \left(\frac{en}{k}\right)^k$.
3. $\left(1+\frac{1}{n}\right)^n < \left(1+\frac{1}{n+1}\right)^{n+1}$ and

   $\left(1-\frac{1}{n}\right)^n < \left(1-\frac{1}{n+1}\right)^{n+1}$ for $n \in \mathbb{N}$

Exercise (B)

Problem 3: Prove Stirling's formula

$$\sqrt{2\pi n}\left(\frac{n}{e}\right)^n \leq n! \leq \sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{\frac{1}{12n}}$$

Problem 4: Show that $\binom{n}{k} \leq \frac{1}{\sqrt{2\pi k}}\left(\frac{ne}{k}\right)^k$.

Problem 5: Show that the fun. $f(x) = \binom{x}{n}$ is convex.

Exercise (C)

Problem 6: prove that

$c_2\left(\frac{k}{\ln k}\right)^{\frac{3}{2}} < R(4,k) < c_1 k^3$, for some constants c_1 and c_2.

Problem 7: Prove that $R(4,k) \geq c\left(\frac{k}{\ln k}\right)^2$ for some constant c.

Problem 8: Show that $c\left(\frac{k}{\ln k}\right)^2 = k^{2+o(1)}$.

Exercise (D)

Problem 9: show that $\sqrt{2} \leq \left(R(k,k)\right)^{\frac{1}{k}} \leq 4.$