

Final Examination (只有二題)

- ※ 請在 1 月 16 日晚上 12 點前 e-mail 給我期末考答案卷。
- ※ 期中考該繳交的資料也請在 1 月 16 日晚上 12 點前給我。
- ※ 期末考答案卷內使用的重要定理也請一併列出證明，答案卷得分的高低乃根據解題詳細狀況而定。

Definition we say that a row vector $v = (v_1, v_2, v_2, \dots, v_t)$ is a *distribution vector* if $v_1, v_2, v_2, \dots, v_t \geq 0$ and $v_1 + v_2 + v_2 + \dots + v_t = 1$

Problem 1. Let G be a **non-bipartite connected graph** with n vertices and m edges, and $\{X_0, X_1, X_2, \dots\}$ be a random walk on G . Show that the distribution of X_k tends to a **stationary distribution** of the transition matrix of the random walk.

Problem 2. Let $P = [p_{ij}]$ be a **transition matrix** of a Markov chain which has state space S . Suppose that $p_{ij} > 0$ for each i, j in state space S . Show that, for any distribution vector v , P has a unique stationary distribution \bar{v} which is independent of v , such that

$$\lim_{r \rightarrow \infty} vP^r = \bar{v} .$$