Exercises (F)

Problem 12: Consider a lattice $(X, \mathcal{P})$. Show that

$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ for all $x, y, z \in X$ **iff**

$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$ for all $x, y, z \in X$.

Problem 13: Let $L \subseteq 2^{[n]}$ with the property that

$A, B \in L \Rightarrow A \cap B \in L$ & $A \cup B \in L$. Show that $(L, \subseteq)$ is a distributive lattice.

ordered by inclusion

Problem 14: Let $(L, \mathcal{P})$ be a lattice and $S \subseteq L$.

Show that $\bigvee S = \mathrm{lub}(S)$.

Exercise (G)

Problem 15: Let $(L, \leqslant)$ be a finite distributive lattice. Consider $(D(J(L)), \subseteq)$, show that for any $A, B \in D(J(L))$ we have $A \cup B \in D(J(L))$ and $A \cap B \in D(J(L))$.