Linear Algebra II Quiz 4

Name:____________ Id No.:______________________ Class: __________

Problem 1:(10 points) Let $S = \{\sin t, \cos t, 1, t\}$ and $V = \text{span}(S)$ with the inner product $\langle f, g\rangle = \int_0^\pi f(t)g(t)dt$. Apply the Gram-Schmidt process to the given subset S of V to obtain an orthogonal basis for V. Then normalize the vectors in this basis to obtain an orthonormal basis β for V.

See page 581 in the textbook.

Problem 2:(4 points) Let $S = \{(1, 0, i), (1, 2, 1)\}$ in $\mathbb{C}^3$. Compute $S^\perp$.

See page 581 in the textbook.

Problem 3:(6 points) Let β be a basis for a subspace W of an inner product space V, and let $z \in V$. Prove that $z \in W^\perp$ if and only if $\langle z, v\rangle = 0$ for every $v \in \beta$.

Proof. $\Rightarrow$ Since β is a basis for W. If $v \in \beta$, then $v \in W$. So for $z \in W^\perp$, we have $< z, v >= 0$, for all $v \in \beta$.
$\Leftarrow$ Let $\beta = \{v_1, \cdots, v_n\}$ be a basis for W. For any $u \in W$, u can be written as $u = \sum_{i=1}^n a_i v_i$, where $a_1, \cdots, a_n$ are scalars. Hence, by assumption, we have

$$< z, u >=< z, \sum_{i=1}^n a_i v_i >= \sum_{i=1}^n \overline{a_i} < z, v_i >= 0.$$

This implies that $z \in W^\perp$. □