Advection-enhanced gradient vector flow for active-contour image segmentation

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Outline of the presentation

1. Active-contour image segmentation
2. Gradient vector flow and some improved models
3. Advection-enhanced gradient vector flow model
4. Numerical experiments

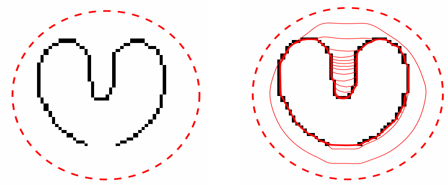
   missing edge recovery, convergence to long and thin indentations, preventing weak-edge leakage, testing on noisy images, real images
5. Summary and conclusions

Active contour (snake)

- The active contour (snake) approach for image segmentation was introduced by *Kass-Witkin-Terzopoulos (IJCV 1988).*
- An active contour is a 2-D parametric curve $z(s) = (x(s), y(s))$, $s \in [0, 1]$, that moves through the spatial domain Ω of an image $I(x, y)$ to minimize the following energy functional:

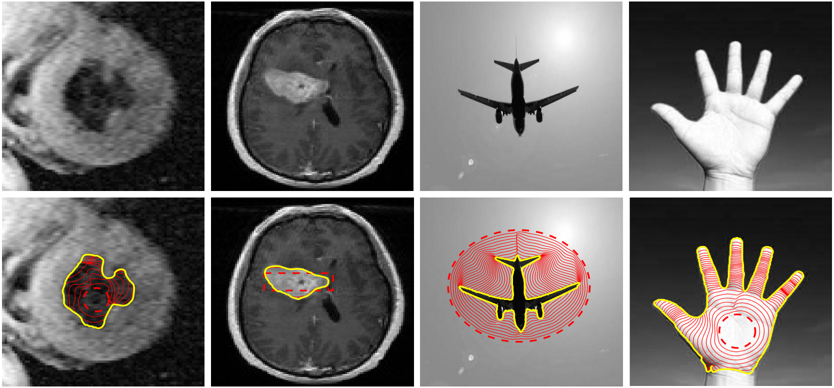
$$\mathcal{E}_{snake}(z) = \int_0^1 \frac{1}{2}\left(\alpha|z_s(s)|^2 + \beta|z_{ss}(s)|^2\right) + E_{ext}(z(s))\, ds.$$

α, β : parameters controlling the snake's tension and rigidity
E_{ext} : a given external function related to the image data



Active contour (snake)

Image segmentation



(Top): Images of cardiac MRI, brain tumor, airplane, and hand
(Bottom): Deformation processes by the proposed AeGVF

An evolution equation

- To obtain a snake $\boldsymbol{z}(s)$ that at least locally minimizes $\mathcal{E}_{snake}$, we consider the associated Euler-Lagrange equation:

$$\underbrace{\alpha \boldsymbol{z}_{ss}(s) - \beta \boldsymbol{z}_{ssss}(s)}_{\boldsymbol{F}_{int}(\boldsymbol{z}(s))} + \underbrace{- \nabla_{\boldsymbol{z}} E_{ext}(\boldsymbol{z}(s))}_{\boldsymbol{F}_{ext}(\boldsymbol{z}(s))} = \boldsymbol{0}, \quad 0 < s < 1.$$

  The internal force $\boldsymbol{F}_{int}$ suppresses the stretching and bending of the snake contour, while the external force $\boldsymbol{F}_{ext}$ attracts it to the desired image features such as the edges.

- In practice, we treat the curve $\boldsymbol{z}$ as a function not only in variable s but also in time t. Thus, we solve the evolution equation,

$$\frac{\partial \boldsymbol{z}}{\partial t}(s,t) - \alpha \boldsymbol{z}_{ss}(s,t) + \beta \boldsymbol{z}_{ssss}(s,t) = \underbrace{- \nabla_{\boldsymbol{z}} E_{ext}(\boldsymbol{z}(s,t)),}_{\text{external force}}$$

  for $(s,t) \in (0,1) \times (0,T]$ to reach a steady-state solution for a time $T \gg 0$, where we have to impose the initial contour $\boldsymbol{z}(s,0) = \boldsymbol{z}_0(s)$ for $s \in [0,1]$ with appropriate BCs.

The external function E_{ext}

- If an image $I(x,y)$ is a line drawing (black on white), the external function can be chosen as one of

$$\begin{aligned} E_{ext}(x,y) &= I(x,y) \quad \text{in } \Omega, \\ E_{ext}(x,y) &= G_\sigma * I(x,y) \quad \text{in } \Omega, \end{aligned}$$

  where G_σ is the two-dimensional Gaussian kernel with standard deviation σ, and the symbol $*$ denotes the usual convolution.

- If an image is a gray-level one and someone wants to seek step edges, two popular external functions are given by

$$\begin{aligned} E_{ext}(x,y) &= -|\nabla I(x,y)|^2 \quad \text{in } \Omega, \\ E_{ext}(x,y) &= -|\nabla(G_\sigma * I(x,y))|^2 \quad \text{in } \Omega. \end{aligned}$$

- **Limitations:** *(i) limited capture range; (ii) poor convergence for concavities.*

The gradient vector flow (GVF) model

- In *Xu-Prince (IEEE-TIP 1998)*, they defined the edge map by
$$f(x,y) := -E_{ext}(x,y) \quad \text{in } \Omega,$$
whose value is larger near the desired features. Then consider the minimization problem: Find $\boldsymbol{V}(x,y) = (u(x,y), v(x,y))^\top$ in a suitable function space that minimizes the energy functional
$$\mathcal{E}(u,v) = \iint_\Omega \mu|\nabla \boldsymbol{V}|^2 + |\nabla f|^2 |\boldsymbol{V} - \nabla f|^2 \, dxdy,$$
where $\mu > 0$ is a regularization parameter and
$$|\nabla \boldsymbol{V}| = |(\nabla u, \nabla v)^\top| = \sqrt{u_x^2 + u_y^2 + v_x^2 + v_y^2}.$$
- Solution $\boldsymbol{V}$ of the minimization problem is called *the gradient vector flow (GVF) field*, which will be applied to replace the term $-\nabla_{\boldsymbol{z}} E_{ext}$ to obtain a snake.

IBVP for the associated Euler-Lagrange equation

- Consider the associated Euler-Lagrange equation. Then the GVF is obtained by solving the IBVP for a time $T \gg 0$ to reach a steady-state solution:

$$\begin{cases} \dfrac{\partial \boldsymbol{V}}{\partial t} = \mu \nabla^2 \boldsymbol{V} - |\nabla f|^2 (\boldsymbol{V} - \nabla f) & \text{in } \Omega \times (0, T], \\ \boldsymbol{V}(\boldsymbol{z}, 0) = \nabla f & \text{in } \Omega, \\ \nabla \boldsymbol{V} \cdot \boldsymbol{n} = \boldsymbol{0} & \text{on } \partial\Omega \times (0, T], \end{cases}$$

  where $\boldsymbol{n}$ denotes the unit outer normal vector to $\partial\Omega$.
- The GVF is almost equal to the external force $\boldsymbol{V} \approx \nabla f = -\nabla E_{ext}$ when $|\nabla f|$ is sufficiently large (near the edges) and *the diffusion term will spread the forces to the regions far from the edges.*
- *We may expect that the GVF has a wider capture range and can enter concave regions.*

Limitations and some improved models

1. If an image has one of the following characteristics, then the GVF snake generally exhibit a poor performance:
   - a weak edge and a strong edge are very close
   - the image has a narrow and deep concavity
   - a strong edge is near a missing edge
   - the image is corrupted by noise
2. We are going to introduce some improved models, including
   - generalized gradient vector flow (GGVF, 1998)
   - normal gradient vector flow (NGVF, 2007)
   - normally biased gradient vector flow (NBGVF, 2010)
   - adaptive diffusion flow (ADF, 2013)
   - *advection-enhanced gradient vector flow (AeGVF)*

The generalized gradient vector flow (GGVF) model

This GGVF is obtained by solving the IBVP at time $T \gg 0$:

$$\begin{cases} \dfrac{\partial \boldsymbol{V}}{\partial t} = g(|\nabla f|)\nabla^2 \boldsymbol{V} - h(|\nabla f|)(\boldsymbol{V} - \nabla f) & \text{in } \Omega \times (0,T], \\ \boldsymbol{V}(\boldsymbol{z},0) = \nabla f & \text{in } \Omega, \\ \nabla \boldsymbol{V} \cdot \boldsymbol{n} = \boldsymbol{0} & \text{on } \partial\Omega \times (0,T], \end{cases}$$

where g and h are spatially varying weighting functions.

- When $g(|\nabla f|) := \mu$ and $h(|\nabla f|) := |\nabla f|^2$, this model reduces to the original GVF model.
- We hope that the effect of diffusion only exists at locations far from the edges to prevent the edges from being polluted too much and that $\boldsymbol{V} \approx \nabla f$ as much as possible when it is near the edges. In *Xu-Prince (Signal Processing 1998)*, they suggested

  $$\begin{aligned} g(|\nabla f|) &:= \exp\{-|\nabla f|/k\}, \\ h(|\nabla f|) &:= 1 - g(|\nabla f|), \end{aligned}$$

  with a constant parameter $k > 0$.

The normal gradient vector flow (NGVF) model

The NGVF is obtained by solving the IBVP at time $T \gg 0$ *(Ning et al., PRL 2007)*:

$$\begin{cases} \dfrac{\partial \boldsymbol{V}}{\partial t} = \mu \boldsymbol{V}_{NN} - |\nabla f|^2 (\boldsymbol{V} - \nabla f) & \text{in } \Omega \times (0, T], \\ \boldsymbol{V}(\boldsymbol{z}, 0) = \nabla f & \text{in } \Omega, \\ \nabla \boldsymbol{V} \cdot \boldsymbol{n} = \boldsymbol{0} & \text{on } \partial\Omega \times (0, T]. \end{cases}$$

The NGVF only employs the normal component $\boldsymbol{V}_{NN}$ to generate the force fields. The Laplacian diffusion can be locally decomposed as

$$\nabla^2 \boldsymbol{V} = \boldsymbol{V}_{TT} + \boldsymbol{V}_{NN},$$

where $\boldsymbol{V}_{TT} = (u_{TT}, v_{TT})^\top$ and $\boldsymbol{V}_{NN} = (u_{NN}, v_{NN})^\top$ denote the second derivatives of $\boldsymbol{V}$ in the tangential direction T and normal direction N.

$$u_{TT} = \frac{u_x^2 u_{yy} + u_y^2 u_{xx} - 2u_x u_y u_{xy}}{|\nabla u|^2}, \quad u_{NN} = \frac{u_x^2 u_{xx} + u_y^2 u_{yy} + 2u_x u_y u_{xy}}{|\nabla u|^2}.$$

It is an anisotropic method and the attractive forces on the bottom of the concavities have more chances to be spread out to the entrance.

The normally biased gradient vector flow (NBGVF) model

The NBGVF is obtained by solving the IBVP at time $T \gg 0$ *(Wang et al., IEEE-SPL 2010)*:

$$\begin{cases} \dfrac{\partial \boldsymbol{V}}{\partial t} & = & \mu\left(\boldsymbol{V}_{TT} + g(|\nabla f|)\boldsymbol{V}_{NN}\right) - |\nabla f|^2(\boldsymbol{V} - \nabla f) \quad \text{in } \Omega \times (0, T], \\ \boldsymbol{V}(\boldsymbol{z}, 0) & = & \nabla f \quad \text{in } \Omega, \\ \nabla \boldsymbol{V} \cdot \boldsymbol{n} & = & \boldsymbol{0} \quad \text{on } \partial\Omega \times (0, T], \end{cases}$$

where $g(|\nabla f|) := \exp\{-(|\nabla f|/k)^2\}$ with parameter $k > 0$.

- When $|\nabla f|$ is large (i.e., near the edges), this model mainly adopts the diffusion in the tangential direction which is beneficial for preserving weak edges.
- When $|\nabla f|$ is getting small, the value of the bias is getting large so that this model almost uses the Laplacian diffusion to carry forces in homogeneous regions.

An adaptive diffusion flow (ADF) model

The ADF is obtained by solving the IBVP at time $T \gg 0$ *(Wu et al., CVIU 2013)* (expected to possess the advantages of NGVF & NBGVF):

$$\begin{cases} \dfrac{\partial \boldsymbol{V}}{\partial t} = g(|\nabla f|)\Big\{\gamma \dfrac{\Delta_\infty \boldsymbol{V}}{|\nabla \boldsymbol{V}|^2} + (1-\gamma)\nabla\cdot\Big(\dfrac{\Phi'(|\nabla \boldsymbol{V}|)}{|G_\sigma * \nabla \boldsymbol{V}|} G_\sigma * \nabla \boldsymbol{V}\Big)\Big\} \\ \qquad -h(|\nabla f|)\big(\boldsymbol{V} - \nabla f\big) \quad \text{in } \Omega \times (0,T], \\ \boldsymbol{V}(\boldsymbol{z},0) = \nabla f \quad \text{in } \Omega, \qquad \nabla \boldsymbol{V}\cdot \boldsymbol{n} = \boldsymbol{0} \quad \text{on } \partial\Omega \times (0,T], \end{cases}$$

g, h are the same as those of GGVF, Δ_∞ denotes infinity Laplacian,

$$\begin{aligned} \Phi(|\nabla \boldsymbol{V}|) &= \frac{1}{p(|\nabla f|)}\Big(\sqrt{1+|G_\sigma * \nabla \boldsymbol{V}|^2}\Big)^{p(|\nabla f|)}, \\ p(|\nabla f|) &= 1 + \frac{1}{1+|\nabla(G_\sigma * f)|}, \\ \gamma &= \begin{cases} \big(1 - f^2/(5k^2)\big)^2 & \text{if } (f^2/5) \le \ell^2, \\ 0 & \text{otherwise}, \end{cases} \\ \ell &= 1.4826\big(E(|\nabla f - E(|\nabla f|)|\big), \end{aligned}$$

with $k > 0$ a constant parameter and $E(\cdot)$ the mean value.

Advection: an alternative to tangential diffusion

1. In image denoising, it is well known that the total variation (TV) regularization model can suppress the variation caused by noise without penalizing the edge gradient too much.
2. In our recent work, *Hsieh-Shao-Yang (Signal Processing 2018)*, we have introduced an adaptive TV regularizer as a controller to improve the performance of the original TV regularizer:

$$\iint_\Omega \alpha |\nabla w| \, dxdy,$$

   where $\alpha := \alpha(x, y)$ and w can be viewed as either component of the vector field $\boldsymbol{V} = (u, v)^\top$.
3. The associated functional derivative is given by

$$-\nabla \cdot \left(\frac{\alpha}{|\nabla w|} \nabla w \right) = \cdots = -\frac{1}{|\nabla w|} \left(\alpha w_{TT} + \nabla \alpha \cdot \nabla w \right),$$

   which reveals that both the terms αw_{TT} and $\nabla \alpha \cdot \nabla w$ are closely related to the ability for preserving image edges.

The advection-enhanced GVF (AeGVF) model

- Taking $\alpha = f$, the resulting advection term $\nabla f \cdot \nabla w$ can be regarded as a better edge-preserving term than the tangential diffusion term w_{TT}, since it can not only increase the strength of attractive forces to prevent the data fidelity from being destroyed but also leave the homogeneous regions unaffected.
- The AeGVF is obtained by solving the IBVP at time $T \gg 0$:

$$\begin{cases} \dfrac{\partial \boldsymbol{V}}{\partial t} = g(|\nabla f|)\boldsymbol{V}_{NN} - \left(\mu_c \chi_{\Omega \setminus E_c} + \mu_p \chi_{E_p}\right) \nabla f \cdot \nabla \boldsymbol{V} \\ \qquad\quad - h(|\nabla f|)\left(\boldsymbol{V} - \nabla f\right) \quad \text{in } \Omega \times (0, T], \\ \boldsymbol{V}(\boldsymbol{z}, 0) = \nabla f \quad \text{in } \Omega, \\ \nabla \boldsymbol{V} \cdot \boldsymbol{n} = \boldsymbol{0} \quad \text{on } \partial\Omega \times (0, T], \end{cases}$$

where $g(|\nabla f|) := \exp\{-(|\nabla f|/k)^2\}$, $h(|\nabla f|) := 1 - g(|\nabla f|)$, μ_c and μ_p are two nonnegative parameters, χ_A is the characteristic function of set A, E_c and E_p denote the sets of corner points and endpoints of boundary curves in the image, respectively.

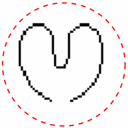
Numerical implementation of snake models

1. We use the explicit forward Euler difference scheme in time and the centered difference scheme in space, so the models with the weighting function g must satisfy the stability condition *(Xu-Prince, Signal Processing 1998)*,

$$\Delta t \leq \frac{\Delta x \Delta y}{4 g_{max}},$$

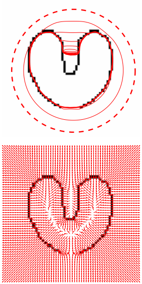
2. Following the idea that the filter G_σ can helpfully suppress the influence of noise, we therefore use $G_\sigma * \boldsymbol{V}$ instead of $\boldsymbol{V}$ in each time step for all improved GVF models, where σ depends on the noise level of an image.

Missing edge recovery #1

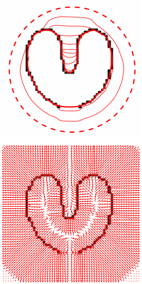


(a) initialization

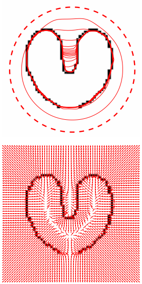
(b) edge map



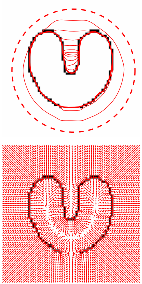
(c) GGVF



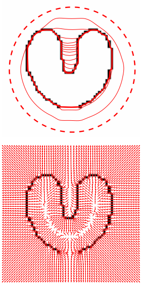
(d) NGVF



(e) NBGVF

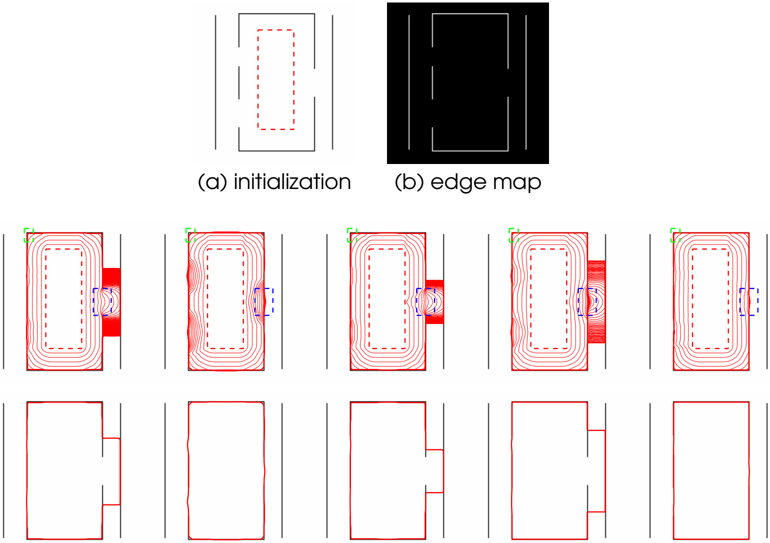


(f) ADF



(g) AeGVF

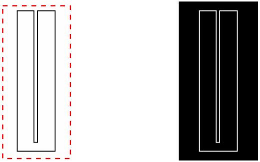
Missing edge recovery #2



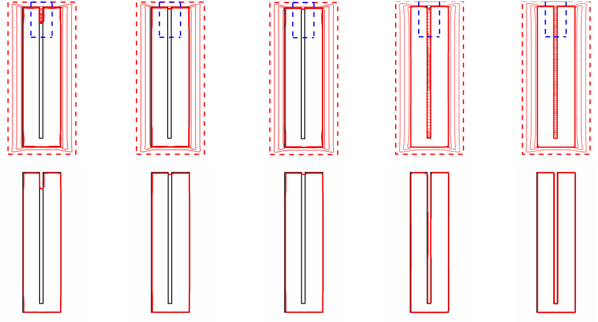
(a) initialization (b) edge map

(c) GGVF (d) NGVF (e) NBGVF (f) ADF (g) AeGVF

Convergence to long and thin indentations #1

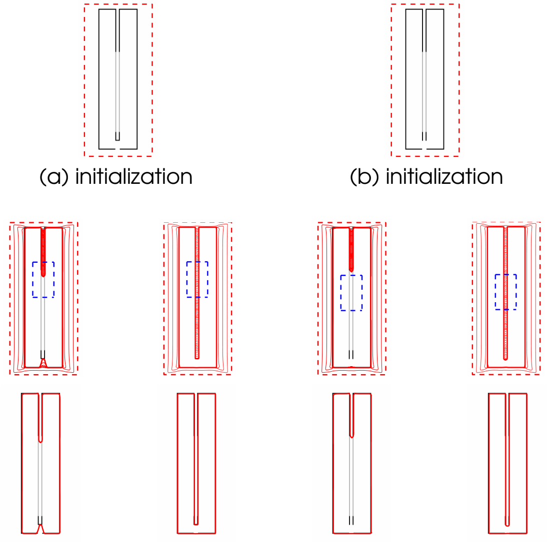


(a) initialization (b) edge map



(c) GGVF (d) NGVF (e) NBGVF (f) ADF (g) AeGVF

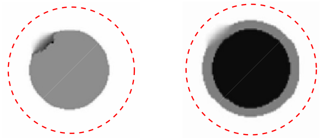
Convergence to long and thin indentations #2



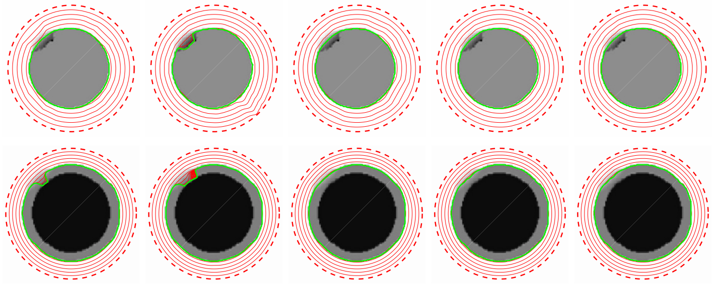
(a) initialization (b) initialization

(c) ADF-550 (d) AeGVF-450 (e) ADF-800 (f) AeGVF-500

Preventing weak-edge leakage #1

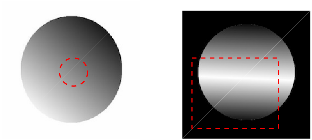


(a) initialization (b) initialization

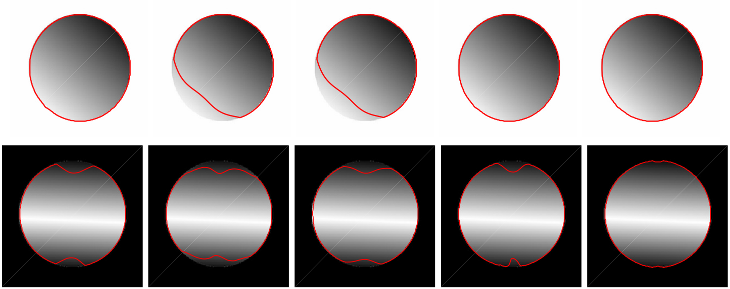


(c) GGVF (d) NGVF (e) NBGVF (f) ADF (g) AeGVF

Preventing weak-edge leakage #2

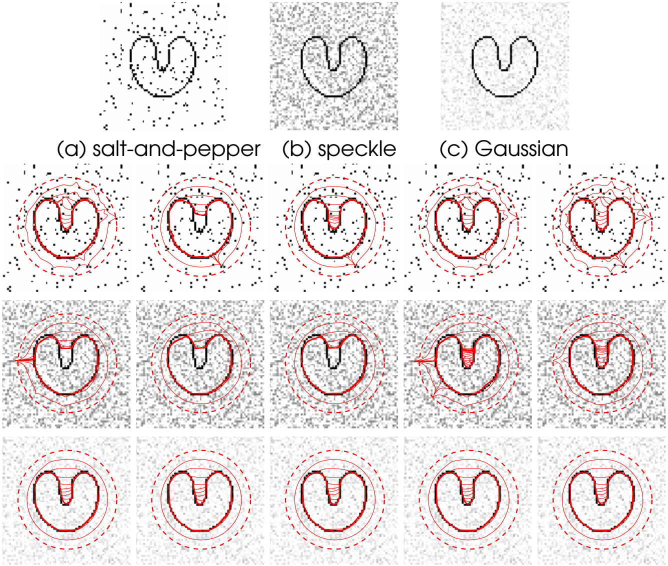


(a) initialization (b) initialization



(c) GGVF (d) NGVF (e) NBGVF (f) ADF (g) AeGVF

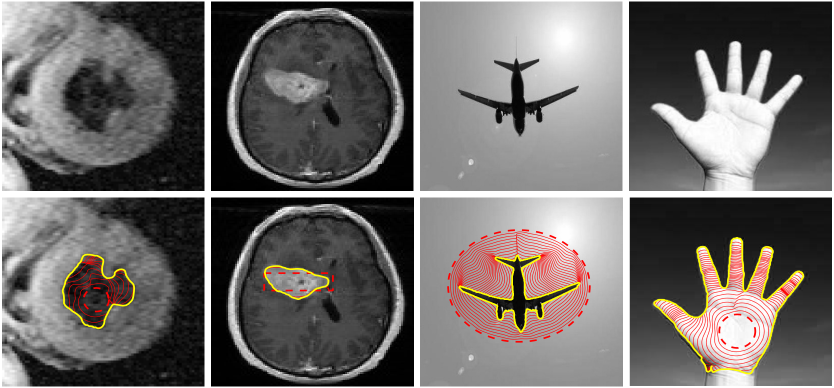
Testing on noisy images



(a) salt-and-pepper (b) speckle (c) Gaussian

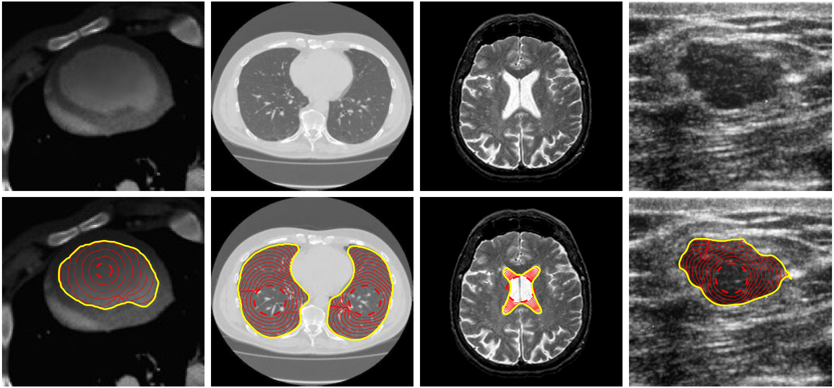
(d) GGVF (e) NGVF (f) NBGVF (g) ADF (h) AeGVF

Image segmentation of some real images



(T): Images of cardiac MRI, brain tumor, airplane, and hand
(B): Deformation processes by the proposed AeGVF

Image segmentation of some real images (cont'd)



(T): Human's cardiac CT, human's lung CT, brain CT, and ultrasound
(B): Deformation processes by the proposed AeGVF

Summary and conclusions

1. We have proposed a new GVF model, called AeGVF, for the active-contour image segmentation. This model is inspired by the functional derivative of an adaptive TV regularizer *(Hsieh-Shao-Yang, Signal Processing 2018).*
2. It is equipped with an advection term in solving the external force field $\boldsymbol{V}$. It has not been investigated before in the literature.
3. The AeGVF snake is able to recover missing edges, to converge to a narrow and deep concavity, and to preserve weak edges. Numerical results show that the AeGVF snake model seems having much better segmentation quality than the others.
4. *A rigorous qualitative analysis of AeGVF model should be very interesting, and this deserves further study.*

References

Details about today's talk can be found in

- Po-Wen Hsieh, Pei-Chiang Shao, and Suh-Yuh Yang, A regularization model with adaptive diffusivity for variational image denoising, *Signal Processing*, 149 (2018), pp. 214-228.
- Po-Wen Hsieh, Pei-Chiang Shao, and Suh-Yuh Yang, Advection-enhanced gradient vector flow for active-contour image segmentation, *submitted for publication*, 2018.

Thank you for your attention!