A mixed H^1-conforming finite element method for Maxwell's equations with non-H^1 solution

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Maxwell's equations

1. Let $\boldsymbol{u} = (u_1, u_2)$ be the electric or magnetic field. The 2-D Maxwell's equations in a simplest form, called the vector potential equations, are given by

$$\nabla \times (\mu^{-1} \nabla \times \boldsymbol{u}) = \boldsymbol{f}, \quad \nabla \cdot (\varepsilon \boldsymbol{u}) = g \text{ in } \Omega, \quad \boldsymbol{u} \cdot \boldsymbol{\tau} = 0 \text{ on } \Gamma,$$

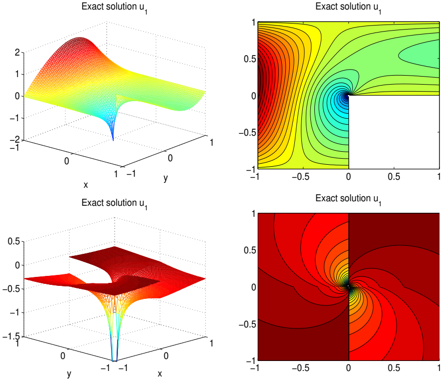
   where $\boldsymbol{\tau}$ denotes the unit tangential vector along boundary Γ of the open bounded domain Ω; $\mu > 0$ and $\varepsilon > 0$ are the magnetic permeability and dielectric permittivity, respectively; and

$$\nabla \times \boldsymbol{u} := \frac{\partial u_2}{\partial x} - \frac{\partial u_1}{\partial y} \quad \text{and} \quad \nabla \times \varphi := \left(\frac{\partial \varphi}{\partial y}, -\frac{\partial \varphi}{\partial x}\right).$$

2. Maxwell's equations are the set of PDEs in terms of curl and divergence operators. In most cases, the regularity of solution is the one of solution of the elliptic problem of Laplacian *minus one.* Therefore, singular solution may appear, $\boldsymbol{u} \in (H^r(\Omega))^2$, $r < 1$.

Singular solutions

Singular solution is typically due to irregular domain boundary and discontinuous, anisotropic, and nonhomogeneous media:



(top) L-domain problem: $\boldsymbol{u} \in (H^{2/3-\delta}(\Omega))^2$

(bottom) discontinuous and nonhomogeneous medium:

$\boldsymbol{u} \in \prod_{j=1}^{4} (H^{\eta}(\Omega_j))^2, \eta < 1/2$

Plain curl/div variational formulation

In this talk, we will focus on the case that the physical domain is non-convex and its boundary includes reentrant corners or edges, which may lead the solution of Maxwell's equations to be a non-H^1 very weak function. In what follows, we set $\mu = \varepsilon = 1$.

1. Recall the Hilbert space for the solution,

$$U = \Big\{ \underbrace{\boldsymbol{v} \in (L^2(\Omega))^2 : \nabla \times \boldsymbol{v} \in L^2(\Omega), \boldsymbol{v} \cdot \boldsymbol{\tau}|_\Gamma = 0,}_{:= H_0(\nabla\times;\, \Omega)} \ \nabla \cdot \boldsymbol{v} \in L^2(\Omega) \Big\},$$

   and the plain curl/div variational formulation,

$$(\nabla \times \boldsymbol{u}, \nabla \times \boldsymbol{v}) + (\nabla \cdot \boldsymbol{u}, \nabla \cdot \boldsymbol{v}) = (\boldsymbol{f}, \boldsymbol{v}) + (g, \nabla \cdot \boldsymbol{v}), \quad \forall\, \boldsymbol{v} \in U.$$

2. **Dirichlet integral:** $\forall\, \boldsymbol{u}, \boldsymbol{v} \in (H^1(\Omega))^2 \cap H_0(\nabla\times;\, \Omega)$, we have

$$(\nabla \times \boldsymbol{u}, \nabla \times \boldsymbol{v}) + (\nabla \cdot \boldsymbol{u}, \nabla \cdot \boldsymbol{v}) = (\nabla \boldsymbol{u}, \nabla \boldsymbol{v}).$$

   This would enforce the nodal-continuous and H^1-conforming finite element solution converging to a member of $H^1(\Omega)$ space, which is an incorrect convergence when $\boldsymbol{u} \in (H^r(\Omega))^2, r \in (0, 1)$.

Some successful nodal-continuous FEMs

The central idea for all these is to modify the plain curl/div weak formulation in either the continuous stage or the discrete stage.

(1) Weighted regularization method (Costabel-Dauge, 2002)

(2) H^{-1} LS method (Bramble-Pasciak, 2004; Badia-Codina, 2012; Bonito-Guermond, 2011)

(3) Weighted dual-potential LS method (Lee-Manteuffel, 2007)

(4) Weighted mixed method (Buffa et al., 2009)

(5) L^2 projection method (Duan et al., 2007, 2009, 2012, 2013, 2014)

H. Y. Duan, R. C. E. Tan, S.-Y. Yang*, and C.-S. You,
A mixed H^1-conforming finite element method for solving Maxwell's equations with non-H^1 solution,
SIAM Journal on Scientific Computing, 40 (2018), pp. A224-A250.

A formulation in mixed form

1. Let Ω be a 2-D nonsmooth domain with re-entrant corners. In this talk, we consider the Maxwell's equations:

$$\nabla \times (\nabla \times \boldsymbol{u}) = \boldsymbol{f}, \quad \nabla \cdot \boldsymbol{u} = g \quad \text{in } \Omega,$$
$$\boldsymbol{u} \cdot \boldsymbol{\tau} = 0 \quad \text{on } \Gamma,$$

where $\boldsymbol{f} \in (L^2(\Omega))^2$ with $\nabla \cdot \boldsymbol{f} = 0$ and $g \in L^2(\Omega)$.

2. Introducing a dummy variable p, the above vector potential equations can be put into the mixed form

$$\nabla \times (\nabla \times \boldsymbol{u}) + \nabla p = \boldsymbol{f}, \quad \nabla \cdot \boldsymbol{u} = g \quad \text{in } \Omega,$$
$$\boldsymbol{u} \cdot \boldsymbol{\tau} = 0, \quad p = 0 \quad \text{on } \Gamma.$$

Note that taking divergence to the first equation, we have

$$\Delta p = 0 \quad \text{in } \Omega, \quad p = 0 \quad \text{on } \Gamma. \quad \Longrightarrow \quad p \equiv 0 \text{ on } \overline{\Omega}.$$

Actually, the role that p plays is the Lagrange multiplier, which is introduced accounting for the divergence constraint $\nabla \cdot \boldsymbol{u} = g$.

A mixed H^1-conforming finite element method

1. A Stokes-like variational problem reads: Find $\boldsymbol{u} \in H_0(\nabla\times;\Omega)$ and $p \in H_0^1(\Omega) := \{q, \frac{\partial q}{\partial x}, \frac{\partial q}{\partial y} \in L^2(\Omega), q|_\Gamma = 0\}$ such that

$$(\nabla \times \boldsymbol{u}, \nabla \times \boldsymbol{v}) + (\nabla p, \boldsymbol{v}) + (\boldsymbol{u}, \nabla q) = (\boldsymbol{f}, \boldsymbol{v}) - (g, q),$$
$$\forall\, \boldsymbol{v} \in H_0(\nabla\times;\Omega),\ q \in H_0^1(\Omega).$$

2. Let $U_h \subset H_0(\nabla\times;\Omega) \cap (H^1(\Omega))^2$ and $Q_h \subset H_0^1(\Omega)$ be the finite element spaces that will be specified later. We propose the stabilized FE method: Find $\boldsymbol{u}_h \in U_h$ and $p_h \in Q_h$ such that

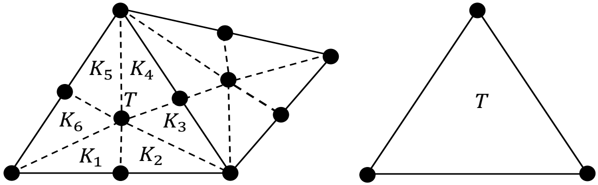
$$(\nabla \times \boldsymbol{u}_h, \nabla \times \boldsymbol{v}_h) + \underbrace{\sum_{T\in\mathcal{T}_h} h_T^2 (\nabla \cdot \boldsymbol{u}_h, \nabla \cdot \boldsymbol{v}_h)_{0,T}}_{\textit{stabilization terms}} + (\nabla p_h, \boldsymbol{v}_h) + (\boldsymbol{u}_h, \nabla q_h)$$

$$= (\boldsymbol{f}, \boldsymbol{v}_h) + \overbrace{\sum_{T\in\mathcal{T}_h} h_T^2 (g, \nabla \cdot \boldsymbol{v}_h)_{0,T}} - (g, q_h), \quad \forall\, \boldsymbol{v}_h \in U_h,\ \forall q_h \in Q_h.$$

Finite element space: CP_1-P_1

Define the CP_1-P_1 finite element spaces U_h and Q_h by

$$\begin{aligned} U_h &= \{ \boldsymbol{v}_h \in H_0(\nabla\times;\Omega) \cap (H^1(\Omega))^2 : \\ &\qquad \boldsymbol{v}_h|_{K_i} \in (P_1(K_i))^2, \forall K_i \subset T, i = 1, 2, \cdots, 6, \forall T \in \mathcal{T}_h \}, \\ Q_h &= \{ q_h \in H_0^1(\Omega) : q_h|_T \in P_1(T), \forall T \in \mathcal{T}_h \}. \end{aligned}$$



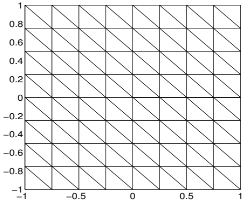
(left) P_1 finite elements for $\boldsymbol{u}_h$ with the Powell-Sabin refinement (incenter), $T = \cup_{i=1}^6 K_i$; (right) P_1 finite elements for p_h.

A smooth solution problem

In order to test the validity of the proposed mixed H^1-conforming FEM, we first consider Maxwell's equations on a square domain $\Omega := (-1,1)^2$ with the smooth exact solution $(\boldsymbol{u}, p) = (u_1, u_2, p)$,

$$u_1(x,y) = \sin(2\pi y)\sin^2(\pi x), \quad u_2(x,y) = \sin(2\pi x)\sin^2(\pi y),$$

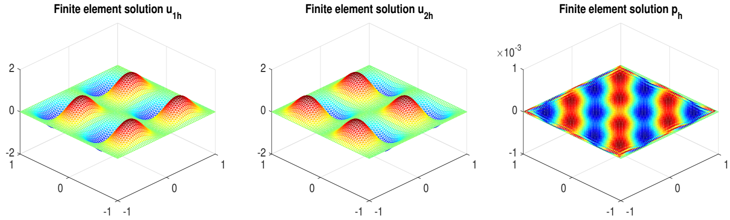
$$p(x,y) = 0.$$



A typical uniform triangle mesh with $h = 1/4$

The smooth solution problem using CP_1-P_1 elements

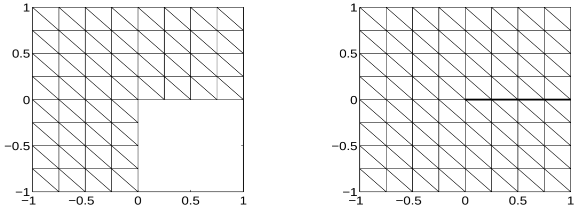
$1/h$	16	32	64	128	256
$\frac{\|\boldsymbol{u}-\boldsymbol{u}_h\|_0}{\|\boldsymbol{u}\|_0}$	3.7489E-03	9.2910E-04	2.3157E-04	5.7825E-05	1.4424E-05
Rate	–	2.01	2.00	2.00	2.00
$\|p-p_h\|_0$	4.6464E-04	3.3096E-05	2.9421E-06	3.1971E-07	2.4014E-08
Rate	–	3.81	3.49	3.20	3.23
$\frac{\|\boldsymbol{u}-\boldsymbol{u}_h\|_{H(\nabla\times;\Omega)}}{\|\boldsymbol{u}\|_{H(\nabla\times;\Omega)}}$	4.3780E-02	2.1890E-02	1.0945E-02	5.4726E-03	2.7363E-03
Rate	–	1.00	1.00	1.00	1.00
$\|p-p_h\|_1$	7.0187E-03	9.3155E-04	1.5419E-04	2.6812E-05	3.3609E-06
Rate	–	2.91	2.59	2.52	3.00



Elevation plots for $h = 1/32$ (regularity $= \infty$)

L-domain and cracked domain problems

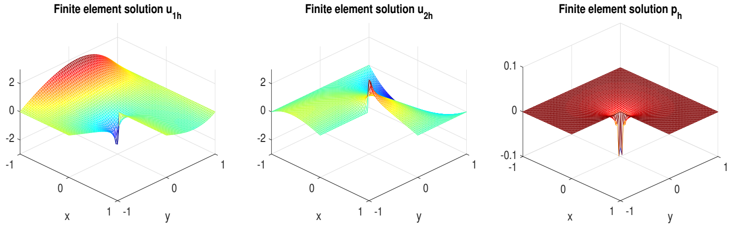
1. **L-domain problem:**
   $\boldsymbol{u}(x,y) = \nabla((1-x^2)(1-y^2)\varphi(x,y))$ and $\varphi(x,y) := \rho^{2/3}\sin(\frac{2}{3}\theta)$
   $\Longrightarrow \boldsymbol{u} \in (H^{2/3-\delta}(\Omega))^2.$
   A strong unbounded singularity appears at O.
2. **Cracked domain problem:**
   $\boldsymbol{u}(x,y) = \nabla((1-x^2)(1-y^2)\varphi(x,y))$ and $\varphi(x,y) := \rho^{1/2}\sin(\frac{1}{2}\theta)$
   $\Longrightarrow \boldsymbol{u} \in (H^{1/2-\delta}(\Omega))^2.$
   A strong unbounded singularity appears at O.



A typical uniform triangle mesh with $h = 1/4$

The L-domain problem using CP_1-P_1 elements

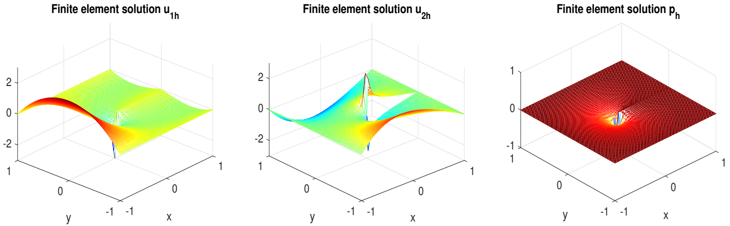
$1/h$	16	32	64	128	256
$\frac{\lVert \boldsymbol{u} - \boldsymbol{u}_h \rVert_0}{\lVert \boldsymbol{u} \rVert_0}$	3.7291E-02	1.9586E-02	1.1093E-02	6.6392E-03	4.0167E-03
Rate	–	0.93	0.82	0.74	0.72
$\lVert p - p_h \rVert_0$	2.1864E-02	9.1305E-03	3.7079E-03	1.4881E-03	5.9407E-04
Rate	–	1.26	1.30	1.32	1.32
$\frac{\lVert \boldsymbol{u} - \boldsymbol{u}_h \rVert_{H(\nabla\times;\Omega)}}{\lVert \boldsymbol{u} \rVert_{H(\nabla\times;\Omega)}}$	5.3043E-02	2.4288E-02	1.2228E-02	6.8028E-03	4.0444E-03
Rate	–	1.13	0.99	0.85	0.75
$\lVert p - p_h \rVert_1$	3.5989E-01	2.3527E-01	1.5021E-01	9.5089E-02	6.0012E-02
Rate	–	0.61	0.65	0.66	0.66



Elevation plots for $h = 1/32$ (regularity ≈ 0.67)

The cracked domain problem using CP_1-P_1 elements

$1/h$	16	32	64	128	256
$\frac{\|\boldsymbol{u}-\boldsymbol{u}_h\|_0}{\|\boldsymbol{u}\|_0}$	3.0383E-01	2.0086E-01	1.2162E-01	7.0411E-02	4.1290E-02
Rate	–	0.60	0.72	0.79	0.77
$\|p-p_h\|_0$	1.7499E-01	1.1456E-01	6.7483E-02	3.6999E-02	1.9419E-02
Rate	–	0.61	0.76	0.87	0.93
$\frac{\|\boldsymbol{u}-\boldsymbol{u}_h\|_{H(\nabla\times;\Omega)}}{\|\boldsymbol{u}\|_{H(\nabla\times;\Omega)}}$	4.3995E-01	2.8772E-01	1.7134E-01	9.6502E-02	5.3910E-02
Rate	–	0.61	0.75	0.83	0.84
$\|p-p_h\|_1$	1.4547E+00	1.3524E+00	1.1279E+00	8.7478E-01	6.6296E-01
Rate	–	0.11	0.26	0.37	0.40

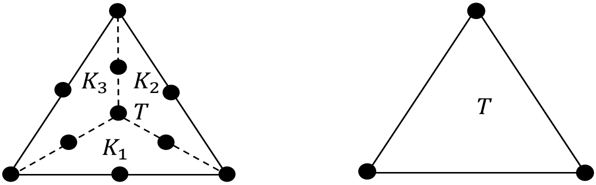


Elevation plots for $h = 1/32$ (regularity ≈ 0.5)

Finite element space: CP_2-P_1

Define the CP_2-P_1 finite element spaces U_h and Q_h by

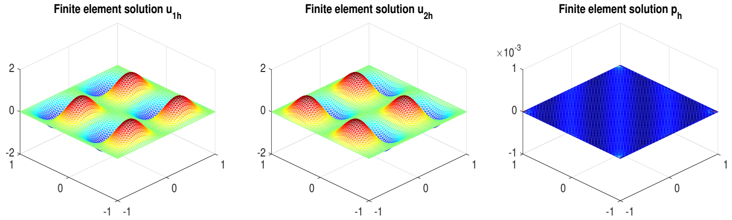
$$
\begin{aligned}
U_h &= \{ \boldsymbol{v}_h \in H_0(\nabla\times;\Omega) \cap (H^1(\Omega))^2 : \\
&\qquad \boldsymbol{v}_h|_{K_i} \in (P_2(K_i))^2, \forall K_i \subset T, i = 1,2,3, \forall T \in \mathcal{T}_h \}, \\
Q_h &= \{ q_h \in H_0^1(\Omega) : \; q_h|_T \in P_1(T), \forall T \in \mathcal{T}_h \}.
\end{aligned}
$$



(left) P_2 finite elements for $\boldsymbol{u}_h$ with the Clough-Tocher refinement (barycenter), $T = \cup_{i=1}^3 K_i$; (right) P_1 finite elements for p_h.

The smooth solution problem using CP_2-P_1 elements

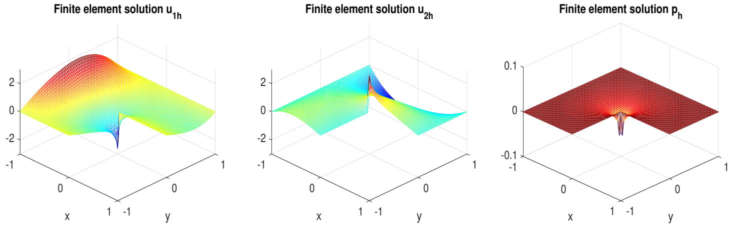
$1/h$	16	32	64	128	256
$\frac{\|\boldsymbol{u}-\boldsymbol{u}_h\|_0}{\|\boldsymbol{u}\|_0}$	1.2525E-03	1.7850E-04	2.3235E-05	2.9375E-06	3.7103E-07
Rate	–	2.81	2.94	2.98	2.99
$\|p-p_h\|_0$	7.3168E-05	1.6152E-06	3.2795E-08	7.5273E-10	2.1786E-11
Rate	–	5.50	5.62	5.45	5.11
$\frac{\|\boldsymbol{u}-\boldsymbol{u}_h\|_{H(\nabla\times;\Omega)}}{\|\boldsymbol{u}\|_{H(\nabla\times;\Omega)}}$	2.9156E-03	7.0103E-04	1.7441E-04	4.3582E-05	1.0877E-05
Rate	–	2.06	2.01	2.00	2.00
$\|p-p_h\|_1$	1.1112E-03	5.4088E-05	3.2996E-06	2.0697E-07	1.3026E-08
Rate	–	4.36	4.03	3.99	3.99



Elevation plots for $h = 1/32$ (regularity $= \infty$)

The L-domain problem using CP_2-P_1 elements

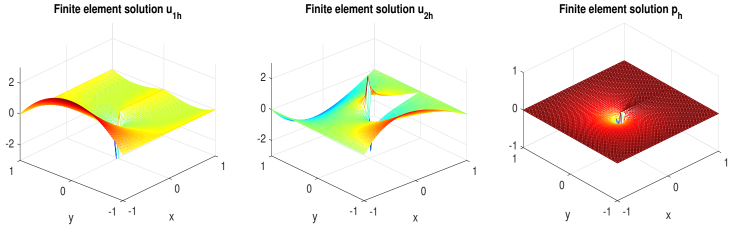
$1/h$	16	32	64	128	256
$\frac{\Vert \boldsymbol{u}-\boldsymbol{u}_h\Vert_0}{\Vert \boldsymbol{u}\Vert_0}$	2.8975E-02	1.6281E-02	9.6914E-03	5.9559E-03	3.7139E-03
Rate	–	0.83	0.75	0.70	0.68
$\Vert p-p_h\Vert_0$	1.3943E-02	5.8502E-03	2.3879E-03	9.6222E-04	3.8520E-04
Rate	–	1.25	1.29	1.31	1.32
$\frac{\Vert \boldsymbol{u}-\boldsymbol{u}_h\Vert_{H(\nabla\times;\Omega)}}{\Vert \boldsymbol{u}\Vert_{H(\nabla\times;\Omega)}}$	3.6256E-02	1.7115E-02	8.9772E-03	5.1533E-03	3.1161E-03
Rate	–	1.08	0.93	0.80	0.73
$\Vert p-p_h\Vert_1$	1.8779E-01	1.2100E-01	7.6870E-02	4.8579E-02	3.0640E-02
Rate	–	0.63	0.65	0.66	0.66



Elevation plots for $h = 1/32$ (regularity ≈ 0.67)

The cracked domain problem using CP_2-P_1 elements

$1/h$	16	32	64	128	256
$\frac{\Vert \boldsymbol{u}-\boldsymbol{u}_h\Vert_0}{\Vert \boldsymbol{u}\Vert_0}$	2.5677E-01	1.6279E-01	9.6385E-02	5.5772E-02	3.2798E-02
Rate	–	0.66	0.76	0.79	0.77
$\Vert p-p_h\Vert_0$	1.4400E-01	9.0576E-02	5.1721E-02	2.7799E-02	1.4436E-02
Rate	–	0.67	0.81	0.90	0.95
$\frac{\Vert \boldsymbol{u}-\boldsymbol{u}_h\Vert_{H(\nabla\times;\Omega)}}{\Vert \boldsymbol{u}\Vert_{H(\nabla\times;\Omega)}}$	3.6847E-01	2.3034E-01	1.3329E-01	7.4336E-02	4.1458E-02
Rate	–	0.68	0.79	0.84	0.84
$\Vert p-p_h\Vert_1$	1.6020E+00	1.4161E+00	1.1397E+00	8.6479E-01	6.3452E-01
Rate	–	0.18	0.31	0.40	0.45



Elevation plots for $h = 1/32$ (regularity ≈ 0.5)

Stability and error estimates for CP_2-P_1 elements

1. *For CP_2-P_1 elements, we have the Babuška-Brezzi inf-sup condition,*

$$\sup_{v_h \in U_h} \frac{b(v_h, q_h)}{|||v_h|||_h} \geq C\|q_h\|_0 \quad \forall\, q_h \in Q_h.$$

2. *Assume that the regular-singular decomposition $\boldsymbol{u} = \boldsymbol{u}^R + \nabla p^S$ holds, where $\boldsymbol{u}^R$ is the regular part, belonging to $(H^{1+r}(\Omega))^2 \cap H_0(\mathrm{curl};\Omega)$, and ∇p^S is the singular part with $p^S \in H^{1+r}(\Omega) \cap H_0^1(\Omega)$, for some $0 \leq r \leq 1$. Then we have*

$$|||\boldsymbol{u} - \boldsymbol{u}_h|||_h + \|p - p_h\|_0 \leq Ch^r \left(\|\boldsymbol{u}^R\|_{1+r} + \|p^S\|_{1+r} + \|\mathrm{div}\,\boldsymbol{u}\|_0\right).$$

3. $|||v_h|||_h^2 := \|\mathrm{curl}\, v_h\|_0^2 + |R_h(\mathrm{div}\, v_h)|_1^2 + \sum_{T \in \mathcal{T}_h} h_T^2 \|\mathrm{div}\, v_h\|_{0,T}^2$ *and we define an H^1 FE projection $R_h : \chi \in H^{-1}(\Omega) \to R_h(\chi) \in Q_h$ by*

$$(R_h(\chi), q_h)_1 := (\nabla R_h(\chi), \nabla q_h) = \langle \chi, q_h \rangle \quad \forall\, q_h \in Q_h.$$

The Maxwell eigenvalue problem

We can further consider the Maxwell eigenvalue problem posed as follows: Find $\omega^2 \in \mathbb{R}$ and $\boldsymbol{u} \in U$ such that

$$\nabla \times (\nabla \times \boldsymbol{u}) = \omega^2 \boldsymbol{u}, \quad \nabla \cdot \boldsymbol{u} = 0 \quad \text{in } \Omega, \quad \boldsymbol{u} \cdot \boldsymbol{\tau} = 0 \quad \text{on } \Gamma.$$

1. The proposed FEM can produce correctly convergent finite element eigenvalues, with convergence rate of $\mathcal{O}(h^{2r})$, whenever the corresponding eigenfunctions are of $(H^r(\Omega))^2, 0 < r < 1$.
2. There are some eigenvalues for which the eigenfunctions belong to $(H^1(\Omega))^2$ and even more regular. For these eigenvalues, the theoretical convergence rate is at most $\mathcal{O}(h^2)$ for CP_1-P_1 and $\mathcal{O}(h^4)$ for CP_2-P_1.

The L-shaped domain eigenvalue problem

The corresponding eigenfunctions are $\boldsymbol{u}^1 \in (H^{2/3-\delta}(\Omega))^2$ *and* $\boldsymbol{u}^2 \in (H^{4/3-\delta}(\Omega))^2$*, for any* $\delta > 0$.

CP_1-P_1 elements

ω^2	$1/h$	ω_h^2	$\lvert\omega^2-\omega_h^2\rvert/\lvert\omega^2\rvert$	Rate
1.47562182408	16	1.94500948373	3.1809E-01	–
	32	1.67437602654	1.3469E-01	1.24
	64	1.55659198774	5.4872E-02	1.30
	128	1.50809033821	2.2003E-02	1.32
	256	1.48855985029	8.7678E-03	1.33
3.53403136678	16	3.53798759701	1.1195E-03	–
	32	3.53473810678	1.9998E-04	2.48
	64	3.53416098758	3.6678E-05	2.45
	128	3.53405625089	7.0413E-06	2.38
	256	3.53403639898	1.4239E-06	2.31

CP_2-P_1 elements

ω^2	$1/h$	ω_h^2	$\lvert\omega^2-\omega_h^2\rvert/\lvert\omega^2\rvert$	Rate
1.47562182408	16	1.77285558616	2.0143E-01	–
	32	1.59849880463	8.3271E-02	1.27
	64	1.52518554833	3.3588E-02	1.31
	128	1.49541776405	1.3415E-02	1.32
	256	1.48349772085	5.3373E-03	1.33
3.53403136678	16	3.53530808872	3.6127E-04	–
	32	3.53423550577	5.7764E-05	2.64
	64	3.53406364542	9.1337E-06	2.66
	128	3.53403645567	1.4400E-06	2.67
	256	3.53403216843	2.2684E-07	2.67

The cracked domain eigenvalue problem

The corresponding eigenfunctions are given by $\boldsymbol{u}^1 \in (H^{1/2-\delta}(\Omega))^2$ *for any* $\delta > 0$ *and* $\boldsymbol{u}^2, \boldsymbol{u}^4 \in (H^1(\Omega))^2$*. The regularity of* $\boldsymbol{u}^3$ *is not known, but our numerical results show that* $\boldsymbol{u}^3 \in (H^{3/2-\delta}(\Omega))^2$*.*

CP_2-P_1 **elements**

ω^2	$1/h$	ω_h^2	$\lvert\omega^2 - \omega_h^2\rvert/\lvert\omega^2\rvert$	Rate
1.03407400850	16	1.89476337739	8.3233E-01	–
	32	1.48919574044	4.4012E-01	0.92
	64	1.26819358462	2.2641E-01	0.96
	128	1.15282009449	1.1483E-01	0.98
2.46740110027	16	2.46740118334	3.3667E-08	–
	32	2.46740110522	2.0082E-09	4.07
	64	2.46740110058	1.2641E-10	3.99
	128	2.46740110027	5.2735E-14	11.23
4.04692529140	16	4.04720868130	7.0026E-05	–
	32	4.04696134468	8.9088E-06	2.97
	64	4.04692981875	1.1187E-06	2.99
	128	4.04692585796	1.4000E-07	3.00
9.86960440109	16	9.86960969039	5.3592E-07	–
	32	9.86960471776	3.2086E-08	4.06
	64	9.86960442060	1.9764E-09	4.02
	128	9.86960440228	1.2051E-10	4.04

Concluding remarks

1. We have proposed a successful mixed H^1-conforming finite element method for solving Maxwell's equations with singular solution in $H^r(\Omega)$, $0 < r < 1$.
2. A pair of H^1-conforming finite elements CP_2-P_1 for electric field $\boldsymbol{u}$ and multiplier p is studied and its stability and error bounds are derived.
3. Numerical experiments for source problems and eigenvalue problems are presented to illustrate the high performance of the proposed method.

References

Details about today's talk can be found in

1. H.-Y. Duan, R. C. E. Tan, S.-Y. Yang*, and C.-S. You,
A mixed H^1-conforming finite element method for solving Maxwell's equations with non-H^1 solution,
SIAM Journal on Scientific Computing, 40 (2018), pp. A224-A250.

2. M. Dauge,
Benchmark computations for Maxwell equations for the approximation of highly singular solutions.
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Thank you for your attention!