Adaptive variational model for contrast enhancement of low-light images

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December 28, 2019 at NCTS

Outline

Contrast enhancement

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Joint work with Po-Wen Hsieh (NCHU) and Suh-Yuh Yang (NCU).

Contrast enhancement

The main purpose of contrast enhancement is to adjust the image intensity to enhance the quality and features of the image for a better human visual perception or machine vision identification.

A low-light image and its enhanced result by our model

Histogram equalization (HE)

- Given a gray-scale image $f : \Omega \to [0,1]$. The cumulative histogram F is defined by considering f as a random variable

$$F(\eta) := \boldsymbol{P}(f \leq \eta) = \frac{1}{|\Omega|}|\{\boldsymbol{x} \in \Omega : f(\boldsymbol{x}) \leq \eta\}|, \quad \eta \in [0,1].$$

- In the second stage, the histogram equalized image is obtained as $F(f(\boldsymbol{x}))$, which is uniformly distributed provided F is invertible

$$\boldsymbol{P}(F(f) \leq \eta) = \boldsymbol{P}(f \leq F^{-1}(\eta)) = F(F^{-1}(\eta)) = \eta.$$

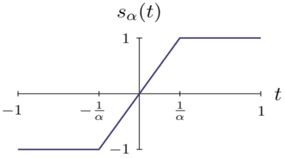
Automatic color equalization (ACE)

- Given a gray-scale image $f : \Omega \to [0,1]$. The following operation is performed

$$\tilde{f}(\boldsymbol{x}) = \sum_{\boldsymbol{y}\in\Omega\setminus\boldsymbol{x}} \frac{s_\alpha(f(\boldsymbol{x}) - f(\boldsymbol{y}))}{\|\boldsymbol{x}-\boldsymbol{y}\|}, \quad \boldsymbol{x}\in\Omega.$$

- In the second stage, $\tilde{f}$ is rescaled to $[0,1]$ as

$$L(\boldsymbol{x}) = (\tilde{f}(\boldsymbol{x}) - \min \tilde{f})/(\max \tilde{f} - \min \tilde{f}).$$



The slope function $s_\alpha(t) := \min\{\max\{\alpha t, -1\}, 1\} \quad (\alpha > 1)$

Variational model for contrast enhancement

- A simple variational model for contrast enhancement was introduced by *Morel-Petro-Sbert (IPOL 2014).*
- Given a gray-scale image $f : \Omega \to \mathbb{R}$, the MPS model is given as

$$\min_u \quad \underbrace{\frac{1}{2}\int_\Omega |\nabla u - \nabla f|^2 \, d\boldsymbol{x}}_{\text{data fidelity term}(D)} \quad + \quad \underbrace{\frac{\lambda}{2}\int_\Omega (u - \bar{u})^2 \, d\boldsymbol{x}.}_{\text{regularization term}(R)}$$

(D): to preserve image details presented in f

(R): to reduce the variance of u to eliminate the effect of nonuniform illumination

$\bar{u}$: the mean value of u over Ω, namely, $\frac{1}{|\Omega|}\int_\Omega u \, d\boldsymbol{x}$.

λ: a positive constant which balances between detail preservation and variance reduction

- Model is simple but very difficult to solve due to the $\bar{u}$ term.

Variational model for contrast enhancement

- They simplified the original model by assuming that the mean value of u coincides with the mean value of f, which gives

$$\min_u \frac{1}{2}\int_\Omega |\nabla u - \nabla f|^2\, d\boldsymbol{x} + \frac{\lambda}{2}\int_\Omega (u-\bar{f})^2\, d\boldsymbol{x}.$$

- Petro-Sbert-Morel (M&AoA 2014) further improved their model by using the L^1 norm to obtain sharper edges.

$$\min_u \int_\Omega |\nabla u - \nabla f|\, d\boldsymbol{x} + \frac{\lambda}{2}\int_\Omega (u-\bar{f})^2\, d\boldsymbol{x}.$$

- Requiring the desired image u being close to a pixel-independent $\bar{f}$ highly contradicts the requirement of ∇u being close to ∇f and restrains the parameter λ to be very small.

The proposed adaptive variational model

- We propose two adaptive functions g and h to replace the pixel-independent constant $\bar{f}$ and the original input image f as

$$\min_u \int_\Omega |\nabla u - \nabla h| \, d\boldsymbol{x} + \frac{\lambda}{2} \int_\Omega (u - g)^2 \, d\boldsymbol{x} + \chi_{[0,255]}(u),$$

where g and h are devised respectively as

$$g(\boldsymbol{x}) := \begin{cases} \alpha \bar{f}, & \boldsymbol{x} \in \Omega_d := \{\boldsymbol{x} \in \Omega : f(\boldsymbol{x}) \leq \bar{f}\}, \\ f(\boldsymbol{x}), & \boldsymbol{x} \in \Omega_b := \{\boldsymbol{x} \in \Omega : f(\boldsymbol{x}) > \bar{f}\}, \end{cases}$$

with a brightness parameter $\alpha > 0$, and

$$h(\boldsymbol{x}) := \begin{cases} \beta f(\boldsymbol{x}), & \boldsymbol{x} \in \Omega_d := \{\boldsymbol{x} \in \Omega : f(\boldsymbol{x}) \leq \bar{f}\}, \\ f(\boldsymbol{x}), & \boldsymbol{x} \in \Omega_b := \{\boldsymbol{x} \in \Omega : f(\boldsymbol{x}) > \bar{f}\}, \end{cases}$$

with a contrast-level parameter $\beta > 1$.

- Ω_d contains relatively dim elements while Ω_b contains relatively bright elements in the image domain Ω.

The proposed adaptive variational model

- The domain division for color RGB images denoted by (f_R, f_G, f_B) is conducted as follows. First, we define the maximum image as

$$f_{\max}(\boldsymbol{x}) := \max\{f_R(\boldsymbol{x}), f_G(\boldsymbol{x}), f_B(\boldsymbol{x})\},$$

  where the max operator is performed pointwisely on each $\boldsymbol{x} \in \Omega$.

- Let $\bar{f}_{\max} := \frac{1}{|\Omega|} \int_\Omega f_{\max}\, d\boldsymbol{x}$.Then we divide the image domain Ω into two parts

$$\begin{aligned} \Omega_d &:= \{\boldsymbol{x} \in \Omega : f_{\max}(\boldsymbol{x}) \le \bar{f}_{\max}\}, \\ \Omega_b &:= \{\boldsymbol{x} \in \Omega : f_{\max}(\boldsymbol{x}) > \bar{f}_{\max}\}. \end{aligned}$$

- As an example, consider an element $\boldsymbol{x}^\star \in \Omega$ with color intensities $(f_R(\boldsymbol{x}^\star), f_G(\boldsymbol{x}^\star), f_B(\boldsymbol{x}^\star)) = (25, 25, 200)$, then $f_{\max}(\boldsymbol{x}^\star) = 200$, a large value which should be classified into Ω_b.

The domain division for color images

(T): Some low-light images (B): The domain-division results

The proposed adaptive variational model

- With the help of the maximum image $f_{\max}$, we can now process color images channelwise. For every $f \in \{f_R, f_G, f_B\}$, we solve

$$\min_u \int_\Omega |\nabla u - \nabla h_c| \, d\boldsymbol{x} + \frac{\lambda}{2} \int_\Omega (u - g_c)^2 \, d\boldsymbol{x} + \chi_{[0,255]}(u),$$

where the adaptive functions g_c and h_c are defined as

$$g_c(\boldsymbol{x}) := \begin{cases} \alpha \bar{f}, & \boldsymbol{x} \in \Omega_d, \\ f(\boldsymbol{x}), & \boldsymbol{x} \in \Omega_b, \end{cases}$$

and

$$h_c(\boldsymbol{x}) := \begin{cases} \beta f(\boldsymbol{x}), & \boldsymbol{x} \in \Omega_d, \\ f(\boldsymbol{x}), & \boldsymbol{x} \in \Omega_b. \end{cases}$$

- There is no evidence shown that chooses different λ, α and β for each channel separately can have specific benefit. Therefore, for simplicity, we fix λ, α, and β across channel in this work.

Existence and uniqueness of minimizer

Definition 1

Let Ω be an open subset of $\mathbb{R}^2$. The space of functions of bounded variation $BV(\Omega)$ is defined as the space of real-valued function $u \in L^1(\Omega)$ such that the total variation

$$\int_\Omega |Du| := \sup\left\{ \int_\Omega u \,\mathrm{div}\varphi \, d\boldsymbol{x} : \varphi \in C_c^1(\Omega, \mathbb{R}^2), \|\varphi\|_{L^\infty(\Omega)} \leq 1 \right\}$$

is finite. Then $BV(\Omega)$ is a Banach space with the norm

$$\|u\|_{BV(\Omega)} := \|u\|_{L^1(\Omega)} + \int_\Omega |Du|.$$

Existence and uniqueness of minimizer

Theorem 1

Let $\Omega \subset \mathbb{R}^2$ be an open bounded domain with Lipschitz boundary and let $h \in BV(\Omega)$ be the input image. Then the variational problem

$$\min_u \int_\Omega |\nabla u - \nabla h| \, d\boldsymbol{x} + \frac{\lambda}{2} \int_\Omega (u-g)^2 \, d\boldsymbol{x} + \chi_{[0,255]}(u)$$

admits a unique minimizer in $BV(\Omega) \cap L^2(\Omega)$.

- $\int_\Omega |\nabla u| \, d\boldsymbol{x}$ should be realized as the total variation $\int_\Omega |Du|$.
- Let $w = u - h$, then the energy can be rewritten as the TV denoising one proposed by Goldstein-Osher (SIIMS 2009).
- Direct method $\longrightarrow$ existence.
- Strict convexity $\longrightarrow$ uniqueness.

The alternating minimization algorithm

- Introducing the discrete gradient operator as $(\nabla u)_{i,j} = ((\nabla_x^+ u)_{i,j}, (\nabla_y^+ u)_{i,j})$ with

$$(\nabla_x^+ u)_{i,j} := \begin{cases} u_{i,j+1} - u_{i,j}, & 1 \le j \le N-1, \\ 0, & j = N, \end{cases}$$

$$(\nabla_y^+ u)_{i,j} := \begin{cases} u_{i+1,j} - u_{i,j}, & 1 \le i \le N-1, \\ 0, & i = N, \end{cases}$$

- Then the continuous model can be discretized as

$$\min_u \sum_{i,j} \left|(\nabla u)_{i,j} - (\nabla f)_{i,j}\right| + \frac{\lambda}{2}(u_{i,j} - g_{i,j})^2 + \chi_{[0,255]}(u).$$

- Applying the operator splitting, it is then equivalent to

$$\min_{u,d,v} \sum_{i,j} \left(|d_{i,j}| + \frac{\lambda}{2}(u_{i,j} - g_{i,j})^2\right) + \chi_S(v), \text{ s.t. } d = \nabla u - \nabla h \;\&\; v = u.$$

The alternating minimization algorithm

- The splitted problem can be solved by using the Bregman iteration (or equivalently the augmented Lagrangian method). Introducing the penalty parameter $\gamma > 0$ and $\delta > 0$, we arrive at the following unconstrained minimization problem

$$\min_{u,d,v} \sum_{i,j} \Big(\big|d_{i,j}\big| + \frac{\lambda}{2}(u_{i,j} - g_{i,j})^2 \tag{1}$$

$$+ \frac{\gamma}{2}\big|d_{i,j} - (\nabla u)_{i,j} + (\nabla h)_{i,j} - b_{i,j}\big|^2 \tag{2}$$

$$+ \frac{\delta}{2}(v_{i,j} - u_{i,j} - c_{i,j})^2 \Big) + \chi_S(v),$$

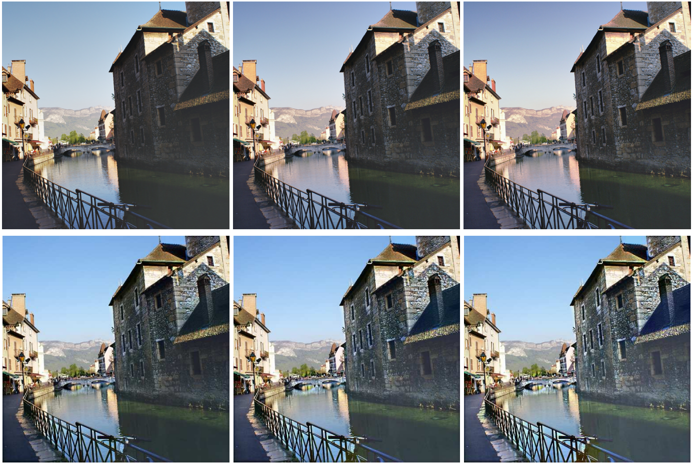
  where b and c are the variables related to the Bregman iteration (or equivalently the Lagrange multipliers).

- Then the problem is solved by alternating the search directions of u, d, and v.

Numerical experiments and comparisons

(T): f, u_{MPS}, u_{HE} (B): u_{VCE}, u_{CLAHE}, $u_{MLHE-HE}$

Numerical experiments and comparisons



(T): $u_{ACE}(\alpha = 2, 4, 6)$ (B): $u_{Adaptive}(\alpha = 0.8, 1.0, 1.2), \beta = 3\alpha)$

Surprisingly, under the same parameter setting, the iteration number of our model is far less than that of the MPS model.

Numerical experiments and comparisons

(T): f, u_{MPS}, u_{HE} (B): u_{VCE}, u_{CLAHE}, $u_{MLHE-HE}$

Numerical experiments and comparisons

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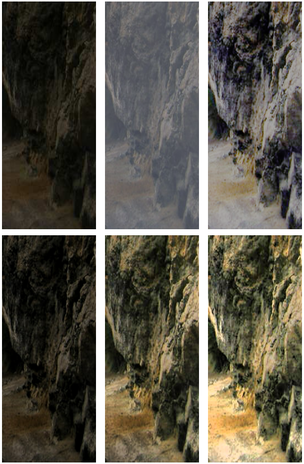

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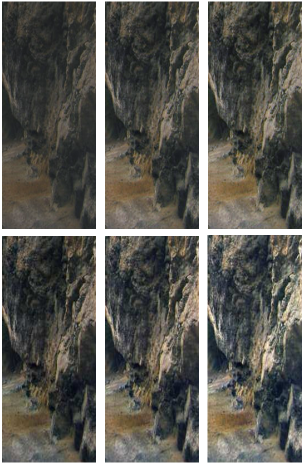
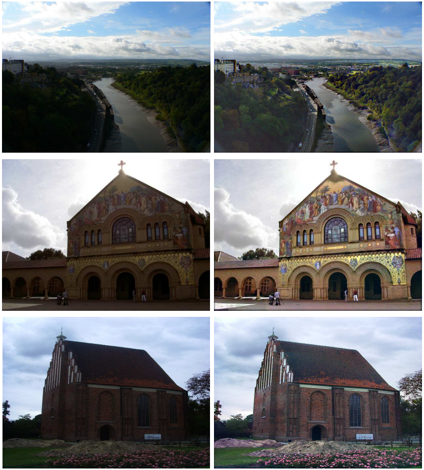
Numerical experiments and comparisons



(T): $u_{ACE}(\alpha = 2, 4, 6)$ (B): $u_{Adaptive}(\alpha = 0.8, 1.0, 1.2, \beta = 3\alpha)$

Other examples



(L): Several low-light images
(R): Enhanced results by the proposed model

Other examples

(L): Several low-light images
(R): Enhanced results by the proposed model

Summary and conclusions

1. We have proposed a simple and efficient adaptive variational model for image contrast enhancement.
2. This model is designed for enhancing low-light images by dividing the image domain into bright and dim parts.
3. The existence and uniqueness of minimizer for the minimization problem is established, and a convergent algorithm is provided.
4. The most distinguished feature of our model is that colors are preserved as close as possible to the original ones.
5. Extending the adaptive idea to other related problems will be our future works.

References

Details about today's talk can be found in

- C. Gatta, A. Rizzi, and D. Marini,
  ACE: An automatic color equalization algorithm, *Proceedings of the First European Conference on Color in Graphics, Image, and Vision (CGIV02)*, 2002, pp. 316-320.
- J.-M. Morel, A. B. Petro, and C. Sbert,
  Screened Poisson equation for image contrast enhancement, *Image Processing On Line*, 4 (2014), pp. 16-29.
- A. B. Petro, C. Sbert, and J.-M. Morel,
  Automatic correction of image intensity non-uniformity by the simplest total variation model, *Methods and Applications of Analysis*, 21 (2014), pp. 91-104.
- P.-W. Hsieh, P.-C. Shao, and S.-Y. Yang,
  Adaptive variational model for contrast enhancement of low light images, *Siam Journal on Imaging Sciences*, 2019.

Thanks for your attention!

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