摘　　要

論文題目：Analysis of synchronization in nonlinearly coupled dynamical networks
(非線性耦合動力網路的同步現象分析)

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論文頁數：87

論文摘要：

在相互影響的大尺度複雜網路系統裡最簡單且著名的集體動力行為就是同步現象。本文主要的目的為探究非線性耦合動力網路中在有或無時間遲滯影響下的全局指數型同步化機制。在沒有時間遲滯影響下，我們引用 Lyapunov 函數結合三種相當不同的技巧取得數個能確保非線性耦合動力網路全局指數型同步的準則。在第一種方法中，我們利用線性矩陣不等式的技巧來研究內含合作-競爭型外耦合矩陣的非對稱非線性耦合動力網路同步現象。第二種方法使用某些圖形理論的技巧，我們改進了 Belykh 等人針對對稱線性耦合動力網路的同步現象分析而發展出所謂的連結圖形穩定法，使其適用於非對稱非線性耦合動力網路。在第三種方法中，我們提出一個以特徵值為基礎的架構，探討隨時間變化的非線性耦合動力網路的同步現象，並進一步以這個方法為基礎探索網路拓樸和網路同步性之間的關係，透過一系列詳盡的數值模擬計算，我們比較了三類不同的網路群在某些相同性質設定下的同步性差異因素。另一方面，針對離散型或連續型時間遲滯影響下的非線性耦合動力網路，我們使用 Lyapunov 泛函結合線性矩陣不等式技術推導出一個使動力網路能全局指數型同步的準則，此同步化準則不僅與時間遲滯大小無關且不受時間遲滯類型的影響。與現存所知文獻的結果相比較，本文主要的優勢在於耦合函數可以是線性或非線性的、狀態變數的分量可以是完全地連結或部分地連結、外耦合矩陣可以是對稱或非對稱的。我們同時提出數個數值模擬實例來驗證上述理論分析的正確性，其中包括了著名的 Chua 耦合電路網、FitzHugh-Nagumo 耦合神經元網路與 Hindmarsh-Rose 耦合神經元網路；文中也特別記述了貓的腦皮層和獼猴的視覺皮層兩個真實神經元網路的數值實驗結果。

關鍵詞：耦合動力網路、離散型時間遲滯、連續型時間遲滯、全局指數型同步、Lyapunov 函數、Lyapunov 泛函、Chua 耦合電路網、FitzHugh-Nagumo 耦合神經元網路、Hindmarsh-Rose 耦合神經元網路、社群網路、小世界網路、無尺度網路。

Abstract

The simplest and most prominent collective behavior in large-scale complex networks of interacting systems is their synchronization. The purpose of this thesis is to investigate the mechanism for global exponential synchronization in nonlinearly coupled dynamical networks without or with coupling time delays. In the absence of coupling time delays, we apply the Lyapunov function method combined with three quite different approaches to derive several criteria that ensure the nonlinearly coupled dynamical networks to be globally exponentially synchronized. First, we study the synchronization in nonlinearly coupled dynamical networks with an asymmetrically cooperative-competitive outer-coupling matrix by utilizing the linear matrix inequality techniques. Secondly, employing some graph theory techniques, we improve the so-called connection graph stability method for the synchronization analysis, that was originally developed by Belykh *et al.* for symmetrically linear coupled dynamical systems, to fit the asymmetrically nonlinear coupled case. Thirdly, we propose a general framework based on an eigenvalue approach for studying the synchronization in time-varying complex networks of nonlinearly coupled dynamical systems. Based on this eigenvalue approach, we explore more deeply the connection between network topologies and network synchronizability. The synchronizability of three network ensembles with prescribed global network properties are compared through a series of numerical computations. On the other hand, for nonlinearly coupled dynamical networks with discrete or distributed time delays, we derive a criterion for the networks to be globally exponentially synchronized from the Lyapunov functional method combined with the linear matrix inequality techniques. This synchronization criterion is independent of not only the time delay but also the delay type. Compared with the existing results in the literature, the primary strengths of this study are that the coupling function can be linear or nonlinear, the components of a state variable can be fully coupled or partially coupled, and the outer-coupling matrix can be symmetric or asymmetrical. Numerical experiments of several illustrative examples including the coupled Chua's circuits, the coupled FitzHugh-Nagumo neurons and the coupled Hindmarsh-Rose neurons are given to demonstrate the theoretical analysis. More interestingly, the numerical results of two real-world networks of the cat cortex and the macaque visual cortex both modeled by the asymmetrically linear coupled FitzHugh-Nagumo equations are also reported.

Keywords: coupled dynamical network; discrete time delay; distributed time delay; global exponential synchronization; Lyapunov function; Lyapunov functional; Chua's circuit; FitzHugh-Nagumo neuron; Hindmarsh-Rose neuron; modular network; small-world network; scale-free network.

AMS subject classifications: 34C15; 34D23; 34K20; 34K23; 92B20.