兩種模擬流構耦合動力行為的沉浸邊界法之研究

摘　　要

本文主要目的為發展高效率的沉浸邊界法用於模擬流構耦合的動力行為,內容可分為下列兩個部份：

在第一部份,我們提出一種新型的懲罰式沉浸邊界法用於模擬一組包含有固體及不可延展界面的史托克流體問題。此方法的主要概念是分別引入三個懲罰式技巧，弱化原始方程組中的流體不可壓縮性、界面不可延展性和固體的剛性運動，這個調整後的統御方程組其優點在於當我們以中央差分在交錯網格上將其離散後,所有未知函數均可與流體速度聯結上關係，使得原本是要處理一個較大的線性代數方程系統，最後只需解決一個跟流體速度有關的較小矩陣問題,特別是當此方法應用到多個複合囊泡問題時將更具優勢與效率，同時我們證明了這小矩陣系統具有對稱負定性質，因此可以使用更高效率的線性求解法。此外，這個新型的懲罰式沉浸邊界法的另一個重要特性是離散格式在適當定義能量泛函下，能量將隨時間遞減，亦即該方法具無條件能量穩定性。數值實驗結果顯示此懲罰式沉浸邊界法在模擬紅血球的動力行為時，具有相當高的精確度與穩定性。

在第二部份，我們提出一個結合投影法的預測-修正型直接施力沉浸邊界法用於模擬流固耦合的動力行為，其中固體可以是靜止或者依指定的速度運動。像往常一樣，流體動力行為是以尤拉座標來描述，而拉格朗日座標則是用來描述固體邊界移動。本文所提出的方法可以被歸類為離散施力法結合預測-修正策略，藉由在流體動量方程式加入離散型的虛擬外力，此虛擬力只在整個固體上作用，以滿足流體在固體邊界上的無滑動邊界條件。更具體地說，基於固體動量變化率，此虛擬力可透過已知的固體速度和由投影法求解不可壓縮納維爾-史托克方程的速度之間的差來預測，此預測虛擬力可進一步加入動量方程再重做一次修正以便更新速度場，壓力和虛擬力。如果有必要的話，這個預測-修正步驟可進行多次迭代，以產生更一般的方法。我們提供一系列標準數值實驗來驗證此方法的簡單性和高效能,我們發現新方法的數值結果與先前在文獻中的結果有很好的一致性，而且在大多數情況下，一個校正步驟就已經足夠。

關鍵詞：不可壓縮納維爾-史托克方程、史托克方程、流構耦合、不可延展界面、固體粒子、沉浸邊界法、懲罰法、直接施力法、投影法、預測-修正、穩定性。

Abstract

This thesis is devoted to developing efficient immersed boundary methods for simulating the dynamics of fluid-structure interaction problems. It is mainly divided into the following two parts:

In the first part, we propose a novel penalty immersed boundary method for simulating the transient Stokes flow with an inextensible interface enclosing a suspended solid particle. The main idea of this approach relies on the penalty techniques by modifying the constitutive equation of Stokes flow to weaken the incompressibility condition, relating the surface divergence to the elastic tension σ to relax the interface's inextensibility, and connecting the particle surface-velocity with the particle surface force $\boldsymbol{F}$ to regularize the particle's rigid motion. The advantage of these regularized governing equations is that when they are discretized by the standard centered difference scheme on a staggered grid, the resulting linear system can easily be reduced by eliminating the unknowns p_h, σ_h and $\boldsymbol{F}_h$ directly, so that we just need to solve a smaller linear system of the velocity approximation $\boldsymbol{u}_h$. This advantage is preserved and even enhanced when such an approach is applied to the transient Stokes flow with multiple compound vesicles. Moreover, this smaller linear system is symmetric and negative-definite, which enables us to use efficient linear solvers. Another important feature of the proposed method is that the discretization scheme is unconditionally stable in the sense that an appropriately defined energy functional associated with the discrete system is decreasing and hence bounded in time. We numerically test the accuracy and stability of the immersed boundary discretization scheme. The tank-treading and tumbling motions of inextensible interface with a suspended solid particle in the simple shear flow will be studied extensively. The simulation of the motion of multiple compound vesicles will be performed as well. Numerical results illustrate the superior performance of the proposed penalty immersed boundary method.

In the second part, we propose a simple prediction-correction direct-forcing immersed boundary method, which is combined with the Choi-Moin projection method, for simulating the dynamics of fluid-solid interaction problems. The immersed solid object can be stationary or moving in the fluid with a prescribed velocity. As usual, an Eulerian description is used for the fluid dynamics, while the Lagrangian representation is employed for the immersed solid boundary. The proposed approach can be categorized as a discrete forcing method with a prediction-correction strategy, in which a virtual force of discrete type distributed on the whole solid body is introduced and added to the fluid momentum equations to accommodate the no-slip boundary condition at the immersed solid boundary. More specifically, based on the rate of moment changes of the solid body, the virtual force at the grid points can be first predicted by using the difference between the prescribed solid velocities and the computed velocities which are obtained by the Choi-Moin projection method for the incompressible Navier-Stokes equations without adding any virtual forcing term. Such predicted virtual force is then put into the momentum equations for the correction step to update the velocity field, pressure and virtual force. This prediction-correction procedure can be iterated to generate a more general method, if necessary. Numerical experiments of several benchmark problems are performed to illustrate the simplicity and high performance of the proposed prediction-correction approach. We find that our numerical results are in very good agreement with the previous works in the literature and in most cases, one correction step is good enough.

Keywords: incompressible Navier-Stokes equations, Stokes equations, fluid-structure interaction, inextensible interface, solid particle, immersed boundary method, penalty method, direct-forcing method, projection method, prediction-correction, stability

AMS subject classifications: 65M06, 65N12, 76D05, 76D07, 76M20