MA 8019: Numerical Analysis I – Homework #2

Name: **Student ID number:**

Consider Wilkinson's polynomial

$$w(x) = \prod_{i=1}^{20}(x-i) = (x-1)(x-2)\cdots(x-20).$$

Clearly, $w(x)$ has 20 zeros located at $x = 1, 2, \cdots, 20$. Expanding the polynomial, one finds

$$\begin{aligned} w(x) \;=\; & x^{20} - 210x^{19} + 20615x^{18} - 1256850x^{17} + 53327946x^{16} \\ & -1672280820x^{15} + 40171771630x^{14} - 756111184500x^{13} \\ & +11310276995381x^{12} - 135585182899530x^{11} \\ & +1307535010540395x^{10} - 10142299865511450x^{9} \\ & +63030812099294896x^{8} - 311333643161390640x^{7} \\ & +1206647803780373360x^{6} - 3599979517947607200x^{5} \\ & +8037811822645051776x^{4} - 12870931245150988800x^{3} \\ & +13803759753640704000x^{2} - 8752948036761600000x \\ & +2432902008176640000. \end{aligned}$$

Let $v(x)$ be the resultant polynomial that the coefficient of x^{19} in $w(x)$ is decreased from -210 to -210.0000001192. Please write a MATLAB code to find the roots of $v(x) = 0$.

From the numerical results, we can find that the problem of approximating the roots of polynomial is ill-conditioned with respect to the coefficients.