MA 8020: Numerical Analysis II
Syllabus and Introduction

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Syllabus

- **Instructor:** Prof. Suh-Yuh Yang (楊肅煜)
  - Office: M315, Hong-Jing Hall
  - Phone: 03-4227151 ext. 65130
- **Office hours**: Tuesday 10:00 $\sim$ 12:00 am or by appointment. No teaching assistant!
- **Prerequisites**: Calculus, Linear Algebra and some knowledge of the software MATLAB: https://portal.ncu.edu.tw/
  校園授權軟體服務網裡面有關於Matlab的下載方式說明！
- **Assignments**: approximately every two weeks, will consist of theoretical problems or computer projects. The students are encouraged to discuss homework with other classmates. *Direct copying is absolutely not allowed.*
- **Examinations:** there will be *a midterm and a final exam.*
- **Grading policy:** *assignments 40%, midterm 30% and final 30%.*

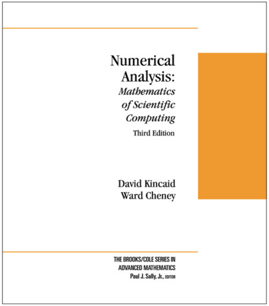
Course objectives

(1) This course introduces students to various types of mathematical analysis that are commonly needed in scientific computing.

(2) The subject of numerical analysis is treated from a mathematical point of view, offering a complete analysis of methods for scientific computing with appropriate motivations and careful proofs.

Textbook

David Kincaid and Ward Cheney, *Numerical Analysis: Mathematics of Scientific Computing, Third Edition,* 2002, Brooks/Cole.



http://www.ma.utexas.edu/CNA/NA3/index.html

Errata: http://www.ma.utexas.edu/CNA/NA3/errata.html

Important dates

- The period for adding and dropping: February 14-29, 2024
- The period for withdrawing: April 01-May 10, 2024
- Peace Memorial Day 2/28 (Wed): recess, no class!
  Spring break 04/03 (Wed): recess, no class!
- Midterm: April 10 (Wed), 2024
- Final exam: June 12 (Wed), 2024

Outline of the course

- Approximating functions (§6.1-§6.4, §6.7-§6.8)
- Numerical differentiation and integration (§7.1-§7.5)
- Numerical solution of ordinary differential equations (§8.1-§8.9, §8.12)
- Numerical solution of partial differential equations (§9.1-§9.4)

Topic 1: Approximating functions

- **Polynomial interpolation:** we are given $n+1$ data points (x_i, y_i), $i = 0, 1, \cdots, n$, and we seek a polynomial p such that $p(x_i) = y_i$, $0 \le i \le n$, where $y_i = f(x_i)$ for some function f.
- **Hermite interpolation:** the interpolation of a function and some of its derivatives at a set of nodes. e.g., find a polynomial p such that $p(x_i) = f(x_i)$ and $p'(x_i) = f'(x_i)$, $i = 0, 1$.
- **Spline interpolation:** a spline function of degree k is a piecewise polynomial of degree at most k having continuous derivatives of all orders up to $k-1$.
- **Taylor series and best approximation**

Topic 2: Numerical differentiation and integration

- **Numerical differentiation**

  – Based on Taylor's theorem: $f(x+h) = f(x) + hf'(x) + \frac{h^2}{2}f''(\xi)$.

  – Based on polynomial interpolation: let p be the Lagrange interpolation of f. Then $f'(x) \approx p'(x)$.

- **Numerical integration based on interpolation:** let p be the Lagrange interpolation of f. Then $\int_a^b f(x)dx \approx \int_a^b p(x)dx$.

- **Gaussian quadrature:** find A_i and x_i, $i = 0, 1, \cdots, n$, such that
$$\int_a^b f(x)dx \approx \sum_{i=0}^{n} A_i f(x_i)$$
and it will be exact for polynomials of degree $\leq 2n+1$.

- **Adaptive quadrature:** the user supplies only f, $[a, b]$ and the accuracy ε desired for computing $\int_a^b f(x)dx$. The program then divides $[a, b]$ into pieces of varying length so that the numerical integration produce results of acceptable precision.

Topic 3: Numerical solution of ordinary differential equations

- **Existence and uniqueness** of the initial value problem:

$$\begin{cases} x'(t) &= f(t,x), \\ x(t_0) &= x_0. \end{cases}$$

- **Taylor-series method:**

$$x(t+h) = x(t) + hx'(t) + \frac{h^2}{2!}x''(t) + \frac{h^3}{3!}x'''(t) + \cdots$$

- **Runge-Kutta methods:** in Taylor-series method, we have to determine $x'', x''', x^{(4)}, \cdots$. RKs avoid this difficulty.
- **Multistep methods:** e.g., the Adams-Bashforth of order 5,

$$x_{n+1} = x_n + \frac{h}{720}\{1901f_n - 2774f_{n-1} + 2616f_{n-2} - 1274f_{n-3} + 251f_{n-4}\}.$$

- **Convergence, stability and consistency:** for multistep method, we have *convergent* $\Longleftrightarrow$ *stable* + *consistent*.
- **Boundary value problems:** shooting method, FDM.

Topic 4: Numerical solution of partial differential equations

- **Parabolic problems:** finite difference method - explicit, implicit.
- **Elliptic problems:** finite difference and finite element methods.
- **Hyperbolic problems:** characteristics.