MA 2007B: Linear Algebra I – Quiz #2

Name: **Student ID number:**

Consider the following linear system

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

(1) (5 pts) Apply elimination and back substitution to find the solution.

Solution:

Elimination:

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \Longleftrightarrow \begin{bmatrix} 2 & -1 & 0 \\ 0 & \frac{3}{2} & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ 1 \end{bmatrix}$$

$$\ell_{21} = \frac{-1}{2},\ \ell_{31} = \frac{0}{2}$$

$$\Longleftrightarrow \begin{bmatrix} 2 & -1 & 0 \\ 0 & \frac{3}{2} & -1 \\ 0 & 0 & \frac{4}{3} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{1}{2} \\ \frac{4}{3} \end{bmatrix}.$$

$$\ell_{32} = \frac{-2}{3}$$

Back substitution:

$$\frac{4}{3}z = \frac{4}{3} \ \Rightarrow\ z = 1 \ \Rightarrow\ \frac{3}{2}y = \frac{1}{2} + z = \frac{3}{2} \ \Rightarrow\ y = 1 \ \Rightarrow\ 2x = 1 + y = 2 \ \Rightarrow\ x = 1.$$

$\therefore$ The solution is $(x, y, z) = (1, 1, 1)$.

(2) (5 pts) Find the three pivots p_1, p_2, p_3 and the three multipliers $\ell_{21}, \ell_{31}, \ell_{32}$.

Solution:

The three pivots are $p_1 = 2,\ p_2 = \frac{3}{2},\ p_3 = \frac{4}{3}$.

The three multipliers are $\ell_{21} = \frac{-1}{2}, \ell_{31} = \frac{0}{2}, \ell_{32} = \frac{-2}{3}$.