MA 2007B: Linear Algebra I – Quiz #3

Name: **Student ID number:**

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(1) (5 pts) Let $\boldsymbol{A}, \boldsymbol{B} \in \mathbb{R}^{n\times n}$. Assume that the product $\boldsymbol{AB}$ is invertible. Show that $\boldsymbol{A}$ is invertible and find the inverse of $\boldsymbol{A}$.

Proof:

$\because$ $\boldsymbol{AB}$ is invertible

$\therefore$ $\exists\, \boldsymbol{C} \in \mathbb{R}^{n\times n}$ such that $(\boldsymbol{AB})\boldsymbol{C} = \boldsymbol{C}(\boldsymbol{AB}) = \boldsymbol{I}$

$\because$ $(\boldsymbol{AB})\boldsymbol{C} = \boldsymbol{A}(\boldsymbol{BC})$

$\therefore$ $\boldsymbol{A}(\boldsymbol{BC}) = \boldsymbol{I}$

$\therefore$ $\boldsymbol{A}$ is is invertible and the inverse of $\boldsymbol{A}$ is $\boldsymbol{A}^{-1} = \boldsymbol{BC}$

(2) (5 pts) Let $\boldsymbol{A} \in \mathbb{R}^{n\times n}$. Assume that there exists a nonzero vector $\boldsymbol{x} \in \mathbb{R}^n$ such that $\boldsymbol{Ax} = \boldsymbol{0}$. Show that $\boldsymbol{A}$ is not invertible.

Proof:

Suppose that $\boldsymbol{A}$ is invertible.

Then $\exists\, \boldsymbol{A}^{-1} \in \mathbb{R}^{n\times n}$ such that $\boldsymbol{AA}^{-1} = \boldsymbol{A}^{-1}\boldsymbol{A} = \boldsymbol{I}$.

$\because$ $\exists\, \boldsymbol{x} \neq \boldsymbol{0}$ such that $\boldsymbol{Ax} = \boldsymbol{0}$

$\therefore$ $\boldsymbol{A}^{-1}\boldsymbol{Ax} = \boldsymbol{A}^{-1}\boldsymbol{0}$

$\therefore$ $\boldsymbol{Ix} = \boldsymbol{0}$

$\therefore$ $\boldsymbol{x} = \boldsymbol{0}$. This is a contradiction!

$\therefore$ $\boldsymbol{A}$ is not invertible