MA 2007B: Linear Algebra I – Quiz #6

Name: **Student ID number:**

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(1) (5 pts) Let $\boldsymbol{A} = [\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n] \in \mathbb{R}^{n\times n}$ be an $n \times n$ invertible real matrix. Show that the column vectors $\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n$ are linearly independent.

Proof:

Let $x_1\boldsymbol{a}_1 + x_2\boldsymbol{a}_2 + \cdots + x_n\boldsymbol{a}_n = \boldsymbol{0}$.

Then $[\boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \boldsymbol{0}$

$\therefore\ \boldsymbol{A}\boldsymbol{x} = \boldsymbol{0}$

$\therefore\ \boldsymbol{A}^{-1}\boldsymbol{A}\boldsymbol{x} = \boldsymbol{A}^{-1}\boldsymbol{0} = \boldsymbol{0}$

$\therefore\ \boldsymbol{x} = \boldsymbol{0}$

$\therefore\ \boldsymbol{a}_1, \boldsymbol{a}_2, \cdots, \boldsymbol{a}_n$ are linearly independent

(2) (5 pts) Let $\boldsymbol{A} \in \mathbb{R}^{m\times n}$ be an $m \times n$ real matrix. Please write down the definitions of the four fundamental subspaces:

The column space of $\boldsymbol{A}$: $C(\boldsymbol{A}) = \{\boldsymbol{A}\boldsymbol{x} |\ \boldsymbol{x} \in \mathbb{R}^n\}$

The row space of $\boldsymbol{A}$: $C(\boldsymbol{A}^\top) = \{\boldsymbol{A}^\top\boldsymbol{y} |\ \boldsymbol{y} \in \mathbb{R}^m\}$

The nullspace of $\boldsymbol{A}$: $N(\boldsymbol{A}) = \{\boldsymbol{x} \in \mathbb{R}^n |\ \boldsymbol{A}\boldsymbol{x} = \boldsymbol{0}\}$

The left nullspace of $\boldsymbol{A}$: $N(\boldsymbol{A}^\top) = \{\boldsymbol{y} \in \mathbb{R}^m |\ \boldsymbol{A}^\top\boldsymbol{y} = \boldsymbol{0}\}$

$$\Big(= \{\boldsymbol{y} \in \mathbb{R}^m |\ \boldsymbol{y}^\top\boldsymbol{A} = \boldsymbol{0}^\top\}\Big)$$