

MA3111: Mathematical Image Processing Syllabus and Introduction



Suh-Yuh Yang (楊肅煜)

Department of Mathematics, National Central University
Jhongli District, Taoyuan City 320317, Taiwan

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Syllabus

- **Instructor:** Prof. Suh-Yuh Yang (楊肅煜)
Office: M315, Hong-Jing Hall
Phone: 03-4227151 extension 65130
- **Office hours:** Tuesday 10:00 ~ 12:00 am or by appointment.
- **Teaching assistant:** 廖育暄
Email: yuhuan2023@g.ncu.edu.tw
- **Prerequisites:** MA1018/MA2030/MA2044, and knowledge of programming language MATLAB
- **Assignments:** will be assigned approximately every two weeks and *announced at ee-class*. The students are encouraged to discuss homework with other classmates. *Direct copying is absolutely not allowed.*
- **Exams:** *there will be a midterm exam and a final presentation/report.*
- **Grading policy:** *assignments 40%, midterm 30% and final 30%.*

Course objective

This course is concerned with the mathematical study of image processing. Its two main objectives are

- (1) *to introduce basic concepts and engineering approaches applicable to digital image processing and develop a further study foundation.*
- (2) *to provide some mathematical techniques for studying several fundamental questions in image processing, such as how to restore a degraded image and how to segment it into meaningful regions.*

No textbook but some references

- (1) [AK2006] G. Aubert and P. Kornprobst, *Mathematical Problems in Image Processing: Partial Differential Equations and the Calculus of Variations, Second Edition*, Springer Verlag, New York, 2006.
- (2) [CS2005] T. F. Chan and J. Shen, *Image Processing and Analysis: Variational, PDE, Wavelet, and Stochastic Methods*, Society for Industrial and Applied Mathematics, Philadelphia, 2005.
- (3) [TUM2019] D. Cremers, *Computer Vision I: Variational Methods, Online Resources*, Departments of Informatics & Mathematics, Technical University of Munich, Germany, 2019/2020.
<https://vision.in.tum.de/teaching/online/cvvm>
- (4) [GW2018] R. C. Gonzalez and R. E. Woods, *Digital Image Processing, Fourth Edition*, Pearson Education Limited, New York, 2018.

Important dates

- The period for adding and dropping: September 4-18, 2024
- The period for withdrawing: October 21-November 29, 2024
- Moon Festival: September 17 (Tue), 2024, *no class!*
- Midterm: November 19 (Tue), 2024 (11th week)
- Sports Day: November 20 (Wed), 2024, *no class!*
- Final presentation/report: December 31 (Tue), 2024 & January 7-8, 2025 (17th & 18th weeks)

Image processing

- Continuous and discrete (digital) images:
 - (1) A grayscale image is a real-valued function $f : \overline{\Omega} \rightarrow \mathbb{R}_0^+$. A color image is a vector-valued function $f : \overline{\Omega} \rightarrow (\mathbb{R}_0^+)^3$.
 - (2) The result of sampling and quantization is a real matrix (矩陣) for a grayscale image and a real tensor (張量) for a color image.
- Image processing is a multidisciplinary field. It overlaps with other areas such as *image analysis/understanding* (影像分析/影像了解) and *computer vision* (電腦視覺).

**image processing $\implies$ image analysis/understanding
 $\implies$ computer vision**

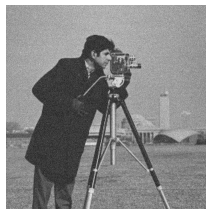
This course will cover the following topics

- (1) Basic concepts of digital image processing
- (2) Intensity transformations and spatial filtering
- (3) Variational methods for image denoising
- (4) Variational methods for image deblurring
- (5) Multi-focus image fusion and guided filter
- (6) Image dehazing: atmospheric scattering model and dark channel prior
- (7) Variational methods for image contrast enhancement
- (8) Image inpainting: variational methods and sparse representation
- (9) Image segmentation: Mumford-Shah and Chan-Vese models
- (10) Principal component pursuit problems

Image denoising



(a) original



(b) noisy



(c) ROF



(d) adaptive

Image deblurring

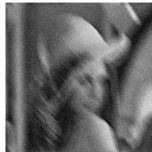
Original image size = 512x512



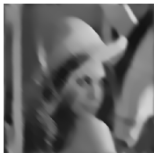
Blurry image (SNR 5.9065)



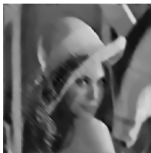
Blurry and noisy image (SNR 5.5328)



$\lambda = 10$ (SNR 7.2649)



$\lambda = 50$ (SNR 9.0242)



$\lambda = 250$ (SNR 10.9436)

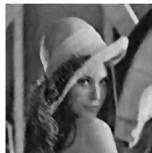


Image fusion



(e) Source1



(f) Source2



(g) LSDGF1



(h) LSDGF2

Image dehazing



(L) hazed images; (R) dehazed images

https://www.cs.huji.ac.il/~raananf/projects/dehaze_cl/results/

Contrast enhancement



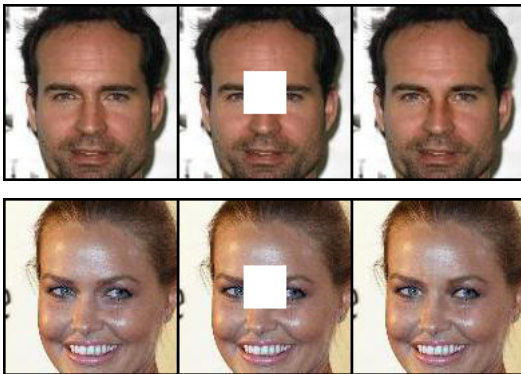
(L) low-light images; (R) contrast enhanced images

Image inpainting I



(L) original images; (M) mask images; (R) inpainted images

Image inpainting II



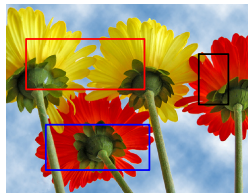
(L) ground truth images; (M) original images; (R) inpainted images

Generative Adversarial Network (GAN, 生成對抗網路)

Image segmentation



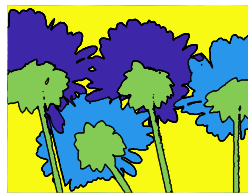
(i)



(j)



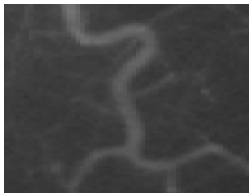
(k)



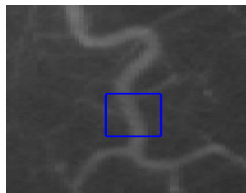
(l)

(i) input image; (j) initial contours; (k)&(l) segmented images

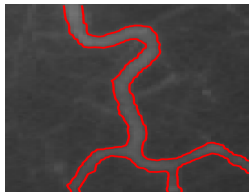
Medical image segmentation (variational method)



(m)



(n)



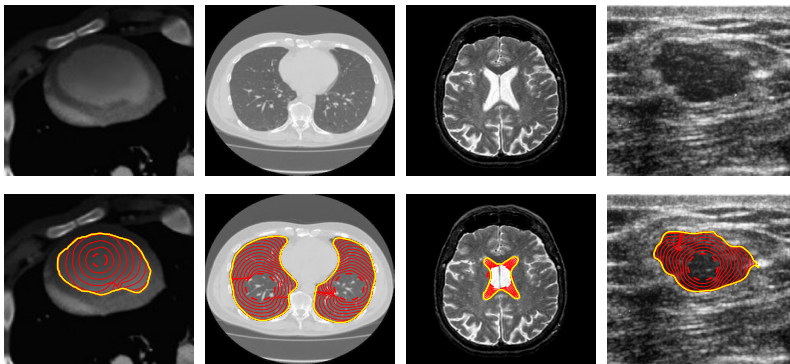
(o)



(p)

(m) input image; (n) initial contour; (o)&(p) segmented images

Medical image segmentation (snake method)



(T) Human's cardiac CT, human's lung CT, brain CT, and ultrasound
(B) Deformation processes by AeGVF

Panorama: image stitching



(T) 8 source images; (B) stitched image