MA 7007: Numerical Solution of Differential Equations I
Syllabus and Introduction

Suh-Yuh Yang (楊肅煜)

Department of Mathematics, National Central University
Jhongli District, Taoyuan City 32001, Taiwan

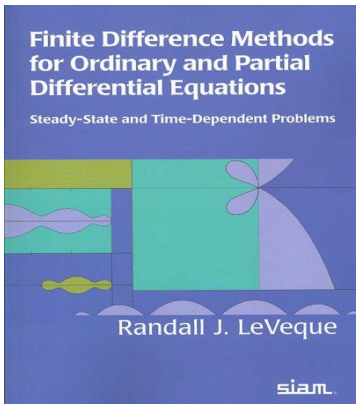
E-mail: syyang@math.ncu.edu.tw
Website: http://www.math.ncu.eud.tw/∼syyang/

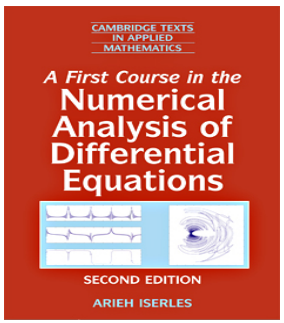
Syllabus

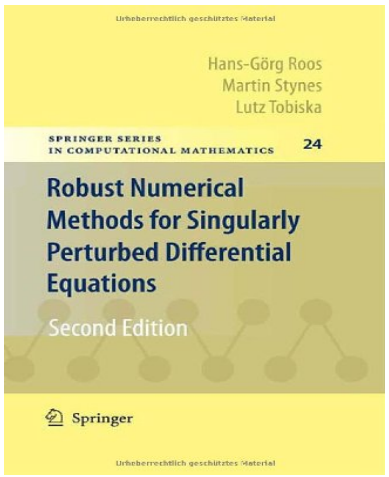
1. **Instructor:** Prof. Suh-Yuh Yang (楊肅煜)
   - Office: M315, Hong-Jing Hall
   - Phone: 03-4227151 ext. 65130
2. **Office hours**: Tuesday 10:00 ∼ 11:50 am or by appointment
3. **Teaching assistant**: 梁長雯, Office: M115, Tel: ext. 65109
   E-mail: treewithout@gmail.com
4. **Prerequisites**: numerical analysis and some knowledge of a high-level programming language Fortran/C/C++, or software package MATLAB: http://matlab.math.ncu.edu.tw/
5. **Assignments**: Approximately every two weeks, will consist of theoretical problems or computer projects. The students are encouraged to discuss homework with other classmates. However, each student needs to turn in her or his own solution set as well as computer codes. Direct copying is absolutely not allowed.
6. **Examinations:** there will be a midterm (Oct. 31) and a final (Dec. 26)
7. **Grading policy:** assignments 40%, midterm 30%, and final 30%

Textbook and references

1. **Textbook:** Randall J. LeVeque, Finite Difference Methods for Ordinary and Partial Differential Equations: Steady State and Time Dependent Problems, SIAM, Philadelphia, July, 2007
2. **References:**
   - Arieh Iserles, A First Course in the Numerical Analysis of Differential Equations, Second Edition, Cambridge University Press, Cambridge, 2009.
   - Hans-Görg Roos, Martin Stynes, and Lutz Tobiska, Robust Numerical Methods for Singularly Perturbed Differential Equations, Second Edition, Springer-Verlag, Berlin, 2008.







Important dates

1. The period for adding and dropping a course: September 06-19, 2017
2. The period for withdrawing a course: October 23-December 08, 2017
3. National Day: October 10, 2017, no class!
4. Midterm: October 31, 2017
5. Final exam: December 26, 2017
6. Last week: January 09, 2018

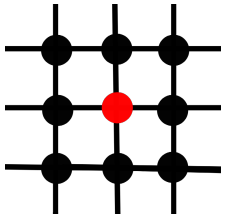
This course will cover the following topics

1. Part I: Boundary Value Problems and Iterative Methods
   - Finite difference approximations
   - Steady states and boundary value problems
   - Elliptic equations
   - Iterative methods for sparse linear systems
2. Part II: Initial Value Problems
   - The initial value problem for ODEs
   - Zero-stability and convergence for initial value problems*
   - Absolute stability for ODEs*
   - Stiff ODEs
   - Diffusion equations and parabolic problems
   - Advection equations and hyperbolic systems

Finite difference method vs. finite element method

1. **Finite difference method:**

$$u(x_i, y_j) \approx u_{ij} \Longrightarrow \boldsymbol{A}\boldsymbol{u} = \boldsymbol{b}$$



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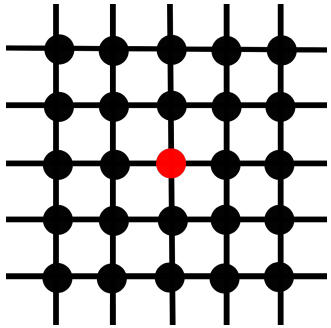

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2. **Finite element method:**

$$u(x,y) \approx u_h(x,y) = x_1\varphi_1(x,y) + x_2\varphi_2(x,y) + \cdots + x_n\varphi_n(x,y)$$
$$\Longrightarrow \boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$$

Scientific computing vs. numerical analysis

1. **Problem modelling**

   physical phenomena: too expensive to perform all tests with prototypes

   mathematical model: (differential or integral equations) too complex or very difficult for paper/pencil solution

   computational model: (numerical methods) approximation of mathematical model

2. **Scientific computing:** solving mathematical problems numerically on the computer (methods/constructive proofs $\rightarrow$ algorithms $\rightarrow$ codes $\rightarrow$ display)

   Numerical analysis – mathematics of scientific computing: it involves the study, development and analysis of algorithms (procedures) for obtaining numerical solutions to various mathematical problems

3. **Scientific computing is an interdisciplinary area of research:** science/engineering; numerical analysis; computer science; software engineering