

NCTS Workshop on Analysis at NCU, 2015

2015/12/28-29 分析研討會議程

12 月 28 日 (星期一)

8:30-8:55 報到

Session 1 主持人：林欽誠

9:00-10:30 韓永生教授〈Auburn Univ, USA〉

奇異積分與 Hardy 空間的乘積理論

10:30-11:00 —Tea Break —

11:00-11:50 王雅書教授〈中興大學〉

Disjointness preserving operators

Session 2 主持人：沈俊嚴

14:00-15:30 韓永生教授〈Auburn Univ, USA〉

Heisenberg 群上的 Marcinkiewz 乘子定理與 flag 奇異積分

15:30-16:00 —Tea Break —

16:00-16:50 陳界山教授〈台灣師範大學〉

Discovery of new complementarity functions for NCP and SOCCP

17:30-20:00 晚餐

12 月 29 日 (星期二)

8:30-8:55 報到

Session 1 主持人：李明憶

9:00-10:30 韓永生教授〈Auburn Univ, USA〉

具有混合齊性的奇異積分及其在相應空間中的有界性 I

10:30-11:00 —Tea Break —

11:00-11:50 黃皓瑋教授〈中山大學〉

Analytic and combinatorial aspects of bi-freely infinitely divisible laws

Session 2 主持人：沈俊嚴

14:00-15:30 韓永生教授〈Auburn Univ, USA〉

具有混合齊性的奇異積分及其在相應空間中的有界性 II

15:30-16:00 —Tea Break—

16:00-16:50 李明憶教授〈中央大學〉

Hardy spaces associated to the Monge-Ampere equations

摘要：

一. 韓永生教授

調和分析開始於上世紀初 Fourier 級數和單複變函數論。上世紀五十年代初，Calderón 和 Zygmund 推廣了 Hilbert 變換，建立了第一代奇異積分理論。該理論為偏微分方程、特別是橢圓形方程提供了強有力的工具。為研究變系數偏微分方程，Calderón 和 Zygmund 發展了第二代奇異積分理論。該理論開發了由 Kohn, Nirenberg 和 Hörmander 隨之而建立的擬微分算子理論。為了引入一類擬微分算子代數，Calderon 考慮了微分算子與乘法算子的交換子，即著名的 Calderón 交換子，從此打開了研究第三代奇異積分理論之門。為了解決出現在多變數中的諸多問題，如次橢圓估計， $\overline{\partial}_b$ 問題以及強擬凸區域等，Stein 多次呼籲發展“多參數分析”。Zygmund 強極大函數和著名的 Marcinkiewz 乘子定理就是早期的多參數分析，即所謂的乘積理論。上世紀八十年代開始發展了乘積奇異積分，Hardy 空間的乘積理論以及乘積 Carleson 測度和乘積 BMO 空間。Stein 指出 “All this leads one to hope that a suitable version of multiparameter analysis will provide the missing theory of singular integrals needed for variety of questions in several complex variable. This is indeed an exciting prospect.”

我們將介紹與多參數分析相關的課題如下：

(一) 奇異積分與 Hardy 空間的乘積理論

八十年代初，Fefferman 和 Stein 建立了乘積奇異積分(卷積)理論，發展了乘積 Littlewood-Paley 理論；Gundy 和 Stein 發展了乘積 H^p 空間；張聖容引進了乘積 Carleson 測度；張聖容和 Fefferman 證明了乘積 H^1 空間的對偶是乘積 BMO 空間；Journé 考慮了一般乘積奇異積分並證明了“T1”定理。利用 Journé 提供的覆蓋引理，Fefferman 證明了乘積奇異積分的 H^p - L^p 有界性。

(二) Heisenberg 群上的 Marcinkiewz 乘子定理與 flag 奇異積分

Heisenberg 群上沒有乘積理論。但是 Müller, Ricci 和 Stein 發現 R^n 上 Marcinkiewz 乘子在 Heisenberg 群上屬於一類新的多參數奇異積分，即 flag 奇異積分，從而證明了 Heisenberg 群上 Marcinkiewz 乘子的 $L^p, 1 < p < \infty$, 有界性。Nagel, Ricci 和 Stein 進一步刻劃了 R^n 上 flag 核並透過 Fourier 變換給

出了 flag 奇異積分和 flag fourier 乘子之間的等價對應。Nagel, Ricci, Stein 和 Wainger 又進一步研究了齊性群上的 flag 奇異積分算子並證明了一類擬局部算子代數。

(三) 具有混合齊性的奇異積分及其在相應空間中的有界性

由第一代奇異積分理論可知，若 $e(x)$ 和 $h(x)$ 分別是 $\mathbb{R}^n$ 中的各向同性和各向異性的零次函數並滿足適當條件，則 Fourier 乘子 $T_1 \widehat{f(x)} = e(x) \hat{f}(x)$ 和 $T_2 \widehat{f(x)} = h(x) \hat{f}(x)$ 是 $L^p, 1 < p < \infty$ 上的有界算子，同時滿足弱(1,1)型估計。Riviere 提出如下問題：複合算子 $T_1 \cdot T_2$ 是否仍然滿足弱(1,1)型估計？Phone 和 Stein 首先回答了該問題並給出 $e(x)$ 和 $h(x)$ 所滿足的充分必要條件。Phone 和 Stein 所考慮的是出現在 $\bar{\partial}$ -Neumann 問題複合算子。最近，Stein 和 Yung 在研究出現在橢圓估計和強擬凸域等問題時，引入了一類混合齊性的擬微分算子。該算子具有兩個參數、即 $S^{m,n}$ 。它同時包含經典的 m 階擬微分算子和 n 階各向異性的擬微分算子。他們證明了 $L^p, 1 < p < \infty$ ，以及 Lipschitz 空間上的有界性，同時還將這些結果推廣到光滑緊流形。

二. 陳界山教授

It is well known that complementarity functions play an important role in dealing with complementarity problems. In this talk, we propose a few new classes of complementarity functions for nonlinear complementarity problems and second-order cone complementarity problems. The constructions of such new complementarity functions are based on discrete generalization which is a novel idea. Surprisingly, the new families of complementarity functions possess continuous differentiability even though they are discrete-oriented extensions. This feature enables that many methods like Newton method can be employed directly for solving nonlinear complementarity problems and second-order cone complementarity problems. This is a new discovery to the literature and we believe that such new complementarity functions can also be used in many other contexts.

三. 李明憶教授

Let ϕ be a strictly convex and smooth function, and $\mu = \det D^2 \phi$ be the Monge-Ampere measure generated by ϕ . For x and $t > 0$, let $S(x, t) := \{y : \phi(y) < \phi(x) + \nabla \phi(x)(y-x) + t\}$ denote the section.

If μ satisfies the doubling property, Caffarelli and Gutierrez (Trans. AMS 348:1075--1092, 1996) provided a variant of the Calderon-Zygmund decomposition and a John-Nirenberg-type inequality associated with sections. Under a stronger uniform continuity condition on μ , they also (Amer. J. Math. 119:423--465, 1997) proved an invariant Harnack's inequality for nonnegative solutions of the Monge-

Ampere equations with respect to sections. The purpose of this paper is to introduce Hardy spaces associated to the sections and the boundedness of the Monge-Ampère singular integral operators.

四. 王雅書教授

A linear operator between function spaces is disjointness preserving if it maps disjoint functions to disjoint functions. Here, two functions are said to be disjoint if at least one of them vanishes at each point. In the study of linear disjointness preserving operators, two questions naturally arise: representation of such operators and whether the inverse of a disjointness preserving linear isomorphism must also be disjointness preserving. In this talk, we will focus on the inverse of a disjointness preserving linear isomorphism between various types of function spaces, including spaces of (little) Lipschitz functions, uniformly continuous functions and differentiable functions. A representation for a general linear disjointness preserving operator on a space of vector-valued C^p functions is also given.

五. 黃皓瑋教授

In free probability, the notion of free convolution of measures has played an important role since its inception by Voiculescu some 30 years ago. Recently, Voiculescu generalized the notion of free independence to simultaneously study left and right actions on reduced free product spaces simultaneously, known as bi-free independence. One generalization of the notion of free convolution of probability distributions on $\mathbb{R}$ to the bi-free setting is the bi-free convolution of probability distributions on $\mathbb{R}^2$. In this talk, we will explain that the bi-freely infinitely divisible laws, and only these laws, can be used to approximate the distributions of sums of identically distributed bi-free pairs of commuting faces. We will also talk about bi-free convolution semigroups of measures and their Levy-Khintchine representations from an infinitesimal point of view. The proofs depend on the bi-free harmonic analysis machinery that we developed for integral transforms of two variables, and the combinatorics of moments and bi-free cumulants. If time permits, some recent developments in this direction will be discussed.