1. (a) Prove: \( \lim_{h \to 0} \frac{\sinh h}{h} = 1 \) and \( \lim_{h \to 0} \frac{\cosh h - 1}{h} = 0 \)

(b) Use above (a) to show:
\[
(\sinh x)' = \cosh x \quad (\cosh x)' = \sinh x \quad \forall x
\]

2. (a) Define \( f(x) = \begin{cases} x\sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases} \)

Show that \( f(x) \) is continuous at \( x = 0 \) but \( f(x) \) does not exist.

(b) Sketch the graph of \( f(x) \)

3. (a) State and prove the Mean Value Theorem (MVT)

(b) Find a point \( c \) satisfying the conclusion of MVT for \( f(x) \) in \( L \)
   
   (i) \( f(x) = \sqrt{x} \quad L = [1, 9] \)
   
   (ii) \( f(x) = \frac{x}{x+1} \quad L = [3, 6] \)

4. (a) State the \((\varepsilon, \delta)\) definition of limit

(b) Use (a) to prove: \( \lim_{x \to 3} \frac{5}{x^2} = \frac{5}{9} \)

5. Show that if \( f \) is a quadratic polynomial, then the midpoint \( c = \frac{a+b}{2} \) satisfies the conclusion of MVT on \([a, b]\)

6. Let \( f(x) = \lfloor \frac{1}{x} \rfloor \) where \( \lfloor \rfloor \) is the greatest integer function

(a) Sketch the graph of \( f(x) \) on \([\frac{1}{2}, 4]\)

(b) Show \( \lim_{x \to 0} x\lfloor \frac{1}{x} \rfloor = 1 \)
7. Show that \( \int_0^\pi f(\sin \theta) \, d\theta = \int_0^1 f(u) \frac{1}{\sqrt{1-u^2}} \, du \)

8. \( \frac{d}{dx} \int_{-3x}^{x^2} \sin^2 t \, dt \)

9. \( \lim_{N \to \infty} \frac{\pi}{2N} \sum_{j=1}^N \sin \left( \frac{\pi j}{3N} \right) \)

10. Find the following integrals respectively

(a) \( \int \frac{2x+1}{x^2+1x+6} \, dx \)

(b) \( \int \cos^2(\alpha v) \sin^2(\alpha v) \, dv \)

(c) \( \int \frac{1}{\sqrt{x^2-4x+18}} \, dx \)

(d) \( \int e^x \cos x \, dx \)

(e) \( \int \frac{\tan^3(\tan x)}{t} \, dt \)

(i) Find the arc length of \( y = \frac{1}{3} x^{3/2} - x^{1/2}, \quad [2, 8] \)

(ii) Find the arc length of \( y = 3x^2, \quad [0, 2] \)

2. Set \( I_m = \int_0^{3/2} \sin^m x \, dx \)

(a) Show that \( I_1 = 1 \) and \( I_2 = \frac{(\sqrt{3})/2}{3} \) and prove that for \( m > 1 \) \( I_m = \left( \frac{m-1}{m} \right) I_{m-2} \)

(b) Show that \( I_3 = \frac{\pi}{3} \) and \( I_4 = \frac{(\sqrt{3})/2}{3} \)

(c) Show more generally:

\[
I_{2m+1} = \frac{x^{2m+1}}{2m+1} - \frac{x^{2m-1}}{2m-1} + \cdots + \frac{x^2}{2} - \frac{x^0}{2} = \frac{1}{3} \int_0^\pi \sin^m x \, dx
\]

(d) Conclude that

\[
\frac{\pi}{3} = \frac{2}{1 \cdot 3}, \quad \frac{4}{3 \cdot 5}, \quad \cdots, \quad \frac{2m-2m}{(2m-1)(2m+1)}, \quad \frac{I_m}{\int_0^{3/2} \sin^m x \, dx}
\]