B.3

1. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 1$, $x = 4$ about the line $y = 1$. Answer= $\frac{7\pi}{6}$.

2. Find the length of the astroid (星形線)

$$x = \cos^3 t,\ y = \sin^3 t,\ 0 \le t \le 2\pi.$$

Answer= 6.

3. Suppose that $\int_1^x f(t)\,dt = e^{-x} + xe^{-x}$. Find $f(x)$.
Answer= $-xe^{-x}$.

4. Let $y = (\ln x)^x$. Find $\frac{dy}{dx}$.

5. Evaluate $\int \frac{dx}{\sqrt{e^{2x}-6}}$.
Answer= $\frac{1}{\sqrt{6}} \sec^{-1}(\frac{e^x}{\sqrt{6}}) + C$

6. Evaluate $\int \sec x dx$.
Answer= $\ln|\sec x + \tan x| + C$.

7. Find $\int \ln x\,dx$.
Answer= $x \ln x - x + C$

8. Evaluate $\int \frac{e^x}{e^{2x}+3e^x+2} dx$.
Answer= $-\ln(e^x + 2) + \ln(1 + e^x)$.

9. Find $\lim_{x\to\infty}(1 - \frac{1}{x})^x$. 請寫出詳細解題過

10. Find the area in the first quadrant (第一象限) that lines under the curve $y = (\ln x)/x^2$ from $x = 1$ to $x = \infty$. 請寫出詳細解題過