B.4

微積分期末考

1. 請使用微積分計算旋轉體體積的方法來算半徑為1 之圓球體積。

2. Find the length of the curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$.
Answer= $\frac{2}{27}(10\sqrt{10} - 1)$. pp.417

3. Suppose that $f(x) = \int_{1/x}^{x} t\,dt$. Find $f'(x)$.
Answer= $\frac{x^4+1}{x^3}$. pp. 392.19

4. Let $y = x^{\ln x}$. Find $\frac{dy}{dx}$.

5. Evaluate $\int \frac{dx}{\sqrt{e^{2x}-6}}$.
Answer= $\frac{1}{\sqrt{6}} \sec^{-1}(\frac{e^x}{\sqrt{6}}) + C$ pp. 483

6. Evaluate $\int \csc x dx$.
Answer= $-\ln|\csc x + \cot x| + C$.

7. Evaluate $\int \frac{1}{1+\cos x}\,dx$.
Answer= $\tan\frac{x}{2}$ pp. 545.58

8. Evaluate $\int \frac{e^x}{e^{2x}+3e^x+2} dx$.
Answer= $-\ln(e^x + 2) + \ln(1 + e^x)$. pp. 564.35

9. Find $\lim_{x\to 0}(\frac{1}{x^x})^x$. 請寫出詳細解題過

10. Find the area in the first quadrant (第一象限) that lines under the curve $y = (\ln x)/x^2$ from $x = 1$ to $x = \infty$. 請寫出詳細解題過