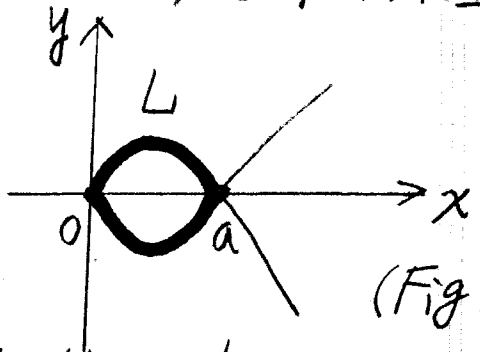
Calculus Test III

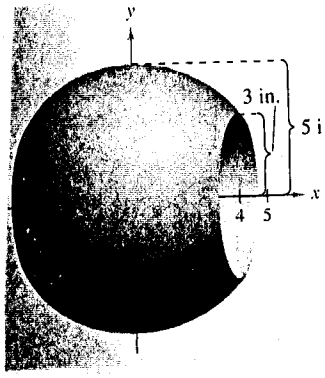
2006/04/21

1. 求曲線 $3ay^2 = x(x-a)^2$ 在 $x \in [0,a]$ 中所圍成封密區域的弧長 L.

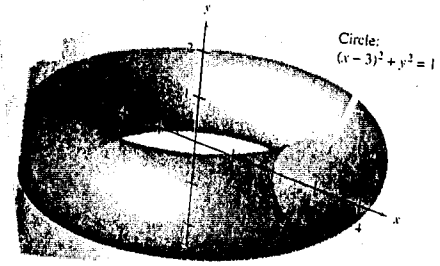


(Fig A)

2. A manufacture drill a holl through the center of a metal sphere of radius 5 inches, as shown in (Fig. B)
The hole has a radius of 3 inches.
What is the volume of the resulting metal ring?



(Fig B)



(Fig C)

3. (a) Find the volume of the torus in (Fig C).
(b) Find the area of the surface of the torus.

4. Find the following integral respectively.

(a) $\int \frac{2x^2+5}{x^2+4}\,dx$ (b) $\int e^{2x}\cos x\,dx$ (c) $\int x^3 e^x\,dx$

(d) $\int \frac{1}{x^2\sqrt{4-x^2}}\,dx$ (e) $\int \sec^3 x\,dx$ (f) $\int_0^{\frac{\pi}{2}} \frac{1}{3-2\cos\theta}\,d\theta$

5. Evaluate the limit:

(a) $\lim_{x\to 0^+} (\sin x)^x$ (b) $\lim_{x\to\infty} \left(1-\frac{5}{x}\right)^x$.