Calculus Final Exam. Total: 120分.

(2006/01/11) (1)~(6)每大題各20% (P1)

(1) Eval the following integral respectively

(a) $\int_0^1 \frac{e^x}{1+e^x}\,dx$ (b) $\int_0^1 x^2 e^x\,dx$

(c) $\int_1^2 x^3 \ln x\,dx$ (d) $\int (\sec t + \tan t)\,dt$

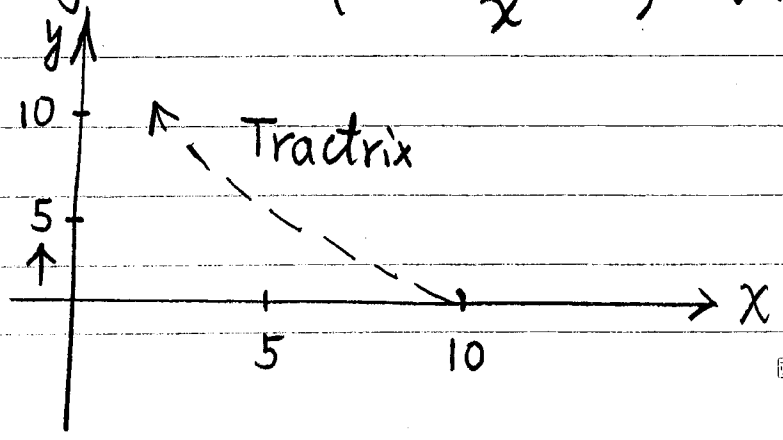
(e) $\int_1^5 \frac{x}{\sqrt{2x-1}}\,dx$ (f) $\int_1^2 x\sqrt{2x-1}\,dx$

(g) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} e^{\sec 2x}\cdot \sec 2x \cdot \tan 2x\,dx$ (h) $\int \frac{x^2-3x+2}{x+1}\,dx$

(i) $\int \frac{x^2+2x+3}{x^3+3x^2+9x}\,dx$ (j) $\int \frac{e^{\frac{1}{x^2}}}{x^3}\,dx$

(2) A person walking along a dock drags a boat by a 10-meter rope. The boat travels along a path known as a tractrix (see the following figure). The equation of this path is

$$y = 10\ln\left(\frac{10+\sqrt{100-x^2}}{x}\right) - \sqrt{100-x^2}$$



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(a) What is the slope of this path when $x=5$ and $x=9$?

(b) What does the slope of the path approach as $x \to 10$?

(3) Let $f(x) = \frac{\ln x}{x}$

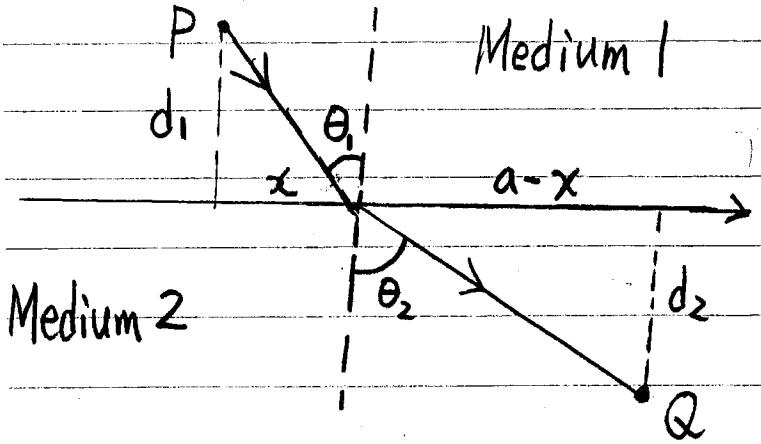
(a) Graph f on $(0, \infty)$ and show that f is strictly decreasing on (e, ∞).

(b) Show that if $e \leq A < B$, then $A^B > B^A$

(c) Use part (b) to obtain that $e^\pi > \pi^e$

4) (Snell's Law of Refraction)

Let θ_1 and θ_2 be the magnitudes of the angle shown in the figure and v_1 and v_2 be the velocities of light in the two media



If the light wave traveling from P to Q follow the path of minimum time, then show : $\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$

(5) Let $f(x) = x^4 - 12x^3 + 48x^2 - 64x$.

Analyze and sketch the graph of f.

(6) (a) Let $f(x) = \int_2^x \sqrt{1+t^2}\,dt$. Find $(f^{-1})'(0) = ?$

(b) Let $f(x) = \cos 2x$, $0 \le x \le \frac{\pi}{2}$.

Find $(f^{-1})'(\frac{\sqrt{2}}{2}) = ?$

(7) Happy New Year !

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