Calculus Exam.

Date : 01. 06. 2004
Total points : 120

P1

(1) (30%) Find the integration and the derivative of the following functions respectively.
   (a) Let $\int_0^1 (6x^5 + 4x^3 + 3x^2 + 2x + 1)\, dx$.
   (b) Let $\int_1^2 (2xe^{x^2} + \frac{5}{x})\, dx$.
   (c) Let $\int_1^2 (1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3})\, dx$.
   (d) $p(x) = (x + (x^2 + 1)^2)^3$, find the equation of the tangent line at the point $(0, 1)$.
   (e) Let $p(x) = xe^{5x^2+2x+6}$. Find $p'(x)$.
   (f) Let $q(x) = \log(x^2 + x + 1) + 3x^2 + 6$. Find $q'(x)$.

(2) (20%) In a research experiment, a population of fruit flies is increasing in accordance with the exponential growth model. After 2 days, there are 100 flies, and after 4 days, there are 300 flies. How many flies will there be after 5 days ?

(3) (20%)A fast-food restaurant has determined that the monthly demand for their hamburgers is given by

$$p = \frac{60000 - x}{20000}.$$

The cost of producing x hamburgers is

$$C = 5000 + 0.56x, 0 \leq x \leq 50000.$$

   (a) Find the increase in revenue per hamburger for monthly sales of 20000 hamburgers, i.e. find the marginal revenue when $x = 20000$.
   (b) Find the profit and the marginal profit when $x = 20000$.
   (c) Find the price p and number x such that the restaurant has the maximal revenue and maximal profit respectively.
   (d) Compare the results in above (c).

(4) (20%) Let $f(x) = 2x^3 - 6x + 4$.
   (a) Draw the graph of the function f.
   (b) Find $\max_{[-1,1]} f(x)$ and $\min_{[-1,1]} f(x)$.

5. (10%) A deposit of \$7500 is made in an account that 6% compounded quarterly. The amount A in the account after t years is

$$A = 7500(1.015)^{[4t]}, t \geq 0.$$

   (a) Sketch the graph of A. Is the graph continuous ? Explain.

(b) Try to find the years during which the finance rate was changing the most and the least respectively.

6. (10%) A deposit of \$7500 is made in an account that 6% compounded quarterly. The amount A in the account after t years is

$$A = 7500(1.015)^{[4t]}, t \geq 0.$$

(a) Sketch the graph of A. Is the graph continuous ? Explain.
(b) What is the balance after 3 years ?