微積分(II) 期末考試 (2002/6/18)

每題10分

Problem 1. Let R be the region in the upper half of the xy plane bounded by the parabolas $y^2 = 4(1-x)$, $y^2 = 4(1+x)$, and the x axis. Compute $\int_R \sqrt{x^2+y^2}dx$ by making the change of variables $x = u^2 - v^2$ and $y = 2uv$.

Problem 2. Use Green's Theorem to find the value $\oint_\lambda ydx + x^2ydy$, where λ is the closed curve formed by $y^2 = x$ and $y = x$ between $(0,0)$ and $(1,1)$.

Problem 3. 求$\int_R x^2y$, 其中R 為$(0,0)$, $(1,2)$, $(2,1)$ 所圍成的三角形。

Problem 4. Convert the following integral to polar coordinates and evaluate. $\int_0^1 \int_{x^2}^{x} (x^2+y^2)^{-1/2}dydx$.

Problem 5. Find the volume of the solid bounded above by the paraboloid $z = 4 - x^2 - y^2$ and below by the plane $z = 4 - 2x$.

Problem 6 Find the volume of the solid bounded by the cylinder $r = 2\cos\theta$, the cone $z = r$, $(r \geq 0)$, and the plane $z = 0$ on a polar coordinate.

Problem 7. Find the area of the region T bounded by the parabolas $y = x^2$ and $y = 4 - x^2$.

Problem 8. Compute the line integral $\int_\lambda (x^2ydx + y^3dy)$, where λ is the arc of the parabola $y = x^2$ from $(0,0)$ to $(1,1)$.

Problem 9. Find the extrema of the function f defined by $f(x,y) = x^3 - 12xy + 8y^2$.

Problem 10. Show that $\int_0^\infty e^{-t^2}dt = \frac{\sqrt{\pi}}{2}$.

Problem 11. 用微分求$\sqrt{27} \times \sqrt[3]{1021}$ 的近似值。

Problem 12. If $f(x,y) = x^2 + xy$ and $P = (1,-1)$, find the maximum value of any directional derivative $D_{\vec{v}}f(P)$.

Problem 13. Define

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Show that f is not continuous at $(0,0)$.

Problem 14. Find the center and radius of curvature of $\lambda(t) = (t^2, t, t^3)$ at $t = 1$.

Problem 15. Find an equation of the plane passing through the three points $(-1, 1, 2)$, $(2, 0, -3)$, and $(5, 1, -2)$.

Problem 16. Find the surface area of a sphere of radius r.

Problem 17. One loop of the lemniscate $r^2 = cos2\theta$ is rotated about the polar axis. Find the area of the surface generated.