

1. $|f(x) - l| < \epsilon$ for all x with $0 < |x - a| < \delta$.
 (or For every $\epsilon > 0$, there exists a corresponding $\delta > 0$ such that for all x , $0 < |x - a| < \delta \Rightarrow |f(x) - l| < \epsilon$.)
2. Arbitrarily give $\epsilon > 0$. $\lim_{x \rightarrow a} f(x) = l$
 $\Rightarrow$ there exists $\delta_1 > 0$ such that
 $(0 < |x - a| < \delta_1 \Rightarrow |f(x) - l| < \frac{\epsilon}{2}) \quad \dots (1)$
 $\lim_{x \rightarrow a} g(x) = m$
 $\Rightarrow$ there exists $\delta_2 > 0$ such that
 $(0 < |x - a| < \delta_2 \Rightarrow |g(x) - m| < \frac{\epsilon}{2}) \quad \dots (2)$
 Take $\delta = \min\{\delta_1, \delta_2\}$, then $0 < |x - a| < \delta$
 $\Rightarrow \begin{cases} 0 < |x - a| < \delta_1 \\ 0 < |x - a| < \delta_2 \end{cases}$
 $\Rightarrow \begin{cases} |f(x) - l| < \frac{\epsilon}{2} & (\because (1)) \\ |g(x) - m| < \frac{\epsilon}{2} & (\because (2)) \end{cases}$
 $\Rightarrow |(f(x) + g(x) - (l + m))|$
 $= |(f(x) - l) + (g(x) - m)|$
 $\leq |f(x) - l| + |g(x) - m| < \epsilon$
 $\therefore$ For an arbitrarily given $\epsilon > 0$, there exist $\delta > 0$ such that $0 < |x - a| < \delta$
 $\Rightarrow |f(x) + g(x) - (l + m)| < \epsilon$.
 Hence $\lim_{x \rightarrow a} f(x) + g(x) = l + m$.