

DIFFERENTIATION

Problems:

1. Show that if f is differentiable at a then f is continuous at a .
2. Give an example of a function f which is continuous at a point a but not differentiable at a .
3. $f(x,y) = \begin{cases} ax^2 + bx + 7 & \text{if } x < 1 \\ 2bx + 2a & \text{if } x \geq 1 \end{cases}$
Suppose that f is differentiable at 1
Find a , b .
4. Find $\frac{dy}{dx}$ for the following:
 - (1) $y = \sin^3(\tan x)$
 - (2) $y = \sec(\cos^{\frac{1}{2}}(x^2 + 5 - \frac{1}{x}))$
 - (3) $y = x^2 \cot(\csc x) + x \sin x \cos x$
 - (4) $y = \sin(x^2 + 1) + \frac{x+2}{x^3+1}$
5. $y = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_3 x^3 + a_2 x^2 + a_1 x + a_0$
Find y' , y'' , $y^{(n)}$ and $y^{(n+1)}$.
6. Find $\frac{dy}{dx}$ if $y^3 = x^2 + \cos \frac{x}{y^2}$.
7. Find $\frac{d^2y}{dx^2}$ if $xy = \sin x + \cos y$.
8. Let C denote the curve $y^4 - 4y^2 = x^4 - 9x^2$.
 - (1) Show that $(3,2)$ and $(3,-2)$ are points on the curve C .
 - (2) Find the equation for the tangent to the curve C at $(3,2)$.
 - (3) Find the equation for the normal to the curve C at $(3,-2)$.
9. Let C be the curve described by

$$x = t - \sin t, \quad y = 1 - \cos t$$

- (1) Find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ for $t = \frac{\pi}{3}$.

- (2) Find the equation for the line tangent to the curve C at the point defined by $t = \frac{\pi}{3}$.
10. A, B are walking on the streets that meet at right angles. A approaches the intersection at 3 m/sec; B moves away from the intersection at 2 m/sec.
At what rate is the angle θ changing and at what rate is the distance between A and B changing when A is 10 m from the intersection and B is 25 m from the intersection?
11. (1) $f(x) = \sqrt[3]{x}$
Find the differential at f .
(2) Use the differential to estimate $\sqrt[3]{1000.2}$.