DIFFERENTUATION

Problems:

1. Show that if f is differentially at a then f is continuous at a.

2. Give an example of a function f which is continuous at a point a but not differentially at a.

3. f(x,y) $\begin{cases} ax^2+bx+7 & \text{if } x<1 \\ 2bx+2a & \text{if } x\geq 1 \end{cases}$
   Suppose that f is differentially at 1
   Find a, b.

4. Find $\frac{dy}{dx}$ for the following:
   (1) $y=\sin^3(\tan x)$
   (2) $y=\sec(\cos^{\frac{1}{2}}(x^2+5-\frac{1}{x}))$
   (3) $y=x^2\cot(\csc x)+x\sin x\cos x$
   (4) $y=\sin(x^2+1)+\frac{x+2}{x^3+1}$

5. $y=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_3x^3+a_2x^2+a_1x+a_0$
   Find $y^1,\ y^2,\ y^{(n)}\ and\ y^{(n+1)}$.

6. Find $\frac{dy}{dx}$ if $y^3=x^2+\cos\frac{x}{y^2}$.

7. Find $\frac{d^2y}{dx^2}$ if $xy=\sin x+\cos y$.

8. Let C denote the curve $y^4-4y^2=x^4-9x^2$.
   (1) Show that (3,2) and (3,-2) are points on the curve C.
   (2) Find the equation for the tangent to the curve C at (3,2).
   (3) Find the equation for the normal to the curve C at (3,-2).

9. Let C be the curve described by

$$x=t-\sin t,\ \ y=1-\cos t$$

   (1) Find $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ for t=$\frac{\pi}{3}$.