

FUNDAMENTAL THEOREM OF CALCULUS

1. State "Fundamental Theorem of Calculus".

2. (1) Suppose that f is a continuous function and g is a differentiable function.

Let

$$H(x) = \int_a^{g(x)} f(t) dt.$$

Find $H'(x)$.

(2) Suppose that f is a continuous function and g_1, g_2 are differentiable functions.

Let

$$K(x) = \int_{g_1(x)}^{g_2(x)} f(t) dt.$$

Find $K'(x)$.

$$3. f(x) = \begin{cases} x + 1 & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ x^2 & \text{if } 1 < x. \end{cases}$$

Let

$$F(x) = \int_{-1}^x f(t) dt.$$

Find $F(x)$ and $F'(x)$.

$$4. f(x) = \begin{cases} x + 1 & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ x^2 + 1 & \text{if } 1 < x. \end{cases}$$

Let

$$F(x) = \int_{-1}^x f(t) dt.$$

Find $F(x)$ and $F'(x)$.