# Exercise Problem Sets 11

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**Problem 1.** Complete the following.

1. Show that if $f : \mathbb{R}^n \to \mathbb{R}^m$ is continuous, and $B \subseteq \mathbb{R}^n$ is bounded, then $f(B)$ is bounded.
2. If $f : \mathbb{R} \to \mathbb{R}$ is continuous and $K \subseteq \mathbb{R}$ is compact, is $f^{-1}(K)$ necessarily compact?

**Problem 2.** Let $(M, d)$ be a metric space, $K \subseteq M$ be compact, and $f : K \to \mathbb{R}$ be lower semi-continuous (see Problem 10 of Exercise 10 for the definition). Show that $f$ attains its minimum on $K$.

**Problem 3. True or False**: Determine whether the following statements are true or false. If it is true, prove it. Otherwise, give a counter-example.

1. Let $f, g$ be a function defined on an open interval. If $f$ and $g$ are functions such that $\lim_{x\to c^+} g(x) = K$, $\lim_{y\to K^+} f(y) = f(K)$, then
$$\lim_{x\to c^+} (f \circ g)(x) = f(K).$$
How about if $x \to c^+$ and $y \to K^+$ are replaced by $x \to c^-$ and $y \to K^-$, respectively?
2. Let $f, g$ be functions defined on an open interval containing $c$ (except possibly at $c$), and $f(x) < g(x)$ for all $x \neq c$. If $\lim_{x\to c} f(x) = L$ and $\lim_{x\to c} g(x) = K$ both exist, then $L < K$.
3. Let $f$ be a function defined on an interval. If $|f|$ is continuous at $c$, so is $f$.
4. Let $I$ be an interval and $f : I \to \mathbb{R}$ be a continuous function. If $f(x) \neq 0$ for all $x \in I$, then $f$ never change signs; that is, either $f(x) > 0$ for all $x \in I$ or $f(x) < 0$ for all $x \in I$.
5. If $\lim_{x\to c} f(x) = \infty$ and $\lim_{x\to c} \big[f(x) - g(x)\big] = 0$, then $\lim_{x\to c} g(x) = \infty$.