

Exercise Problem Sets 2

Sept. 18. 2026

Problem 1. Suppose that $f(x) \neq 0$ for all x near c , and $\lim_{x \rightarrow c} f(x) = 0$. Show that $\lim_{x \rightarrow c} \frac{\sin f(x)}{f(x)} = 1$.

Problem 2. Compute the following limits.

$$(1) \lim_{x \rightarrow 0} \frac{1 - \cos x}{|x|} \qquad (2) \lim_{x \rightarrow 0^+} \frac{\sqrt[3]{1+x} - 1}{1 - \cos \sqrt{x}}.$$
$$(3) \lim_{x \rightarrow 0} \frac{\sin(x^2)}{|x|} \qquad (4) \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+x^2} - 1}{\sin(x^2)}.$$

Problem 3. Show that $\lim_{x \rightarrow 0^+} x^{\frac{3}{4}} \cos \frac{1}{x^2} = 0$ using (1) ε - δ definition and (2) the Squeeze theorem.

Problem 4. 1. Find the limits $\lim_{x \rightarrow 2^+} \frac{x^2 + x - 6}{|x - 2|}$ and $\lim_{x \rightarrow 2^-} \frac{x^2 + x - 6}{|x - 2|}$. Determine whether the limit

$$\lim_{x \rightarrow 2} \frac{x^2 + x - 6}{|x - 2|} \text{ exists or not.}$$

2. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$f(x) = \begin{cases} a + \sin(x - 2) & \text{if } x > 2, \\ x^2 - 3x + b & \text{if } x \leq 2. \end{cases}$$

Find the relation between a and b so that the limit $\lim_{x \rightarrow 2} f(x)$ exists.

3. Let $g : \mathbb{R} \rightarrow \mathbb{R}$ be defined by

$$g(x) = \begin{cases} 1 + \sin(x - 2) & \text{if } x > 2, \\ x^2 - 3x + 3 & \text{if } x \leq 2. \end{cases}$$

Find the limit $\lim_{x \rightarrow 2} \frac{g(x) - 1}{x - 2}$ using the left limit and right limit criteria.

Problem 5. In this problem you are asked to find that limit

$$\lim_{x \rightarrow 0} \frac{\sin(nx) - n \sin x}{x^3}$$

for all natural numbers n . Complete the following.

1. Show that the limit $\lim_{x \rightarrow 0} \frac{\sin(nx) - n \sin x}{x^3}$ exists for all $n \in \mathbb{N}$ by induction.
2. Using the fact that

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} f(2x) \quad \text{whenever one of the limits exists}$$

to find the limit $\lim_{x \rightarrow 0} \frac{\sin(nx) - n \sin x}{x^3}$.

Problem 6. Find the following limits:

1. $\lim_{x \rightarrow -\infty} (2x + \sqrt{4x^2 + 3x - 2})$.
2. $\lim_{x \rightarrow \infty} (x - \sqrt[3]{x^3 + 2x - 3})$.
3. $\lim_{x \rightarrow \infty} \frac{\llbracket x \rrbracket}{x}$, where $\llbracket \cdot \rrbracket$ is the floor function.