Exercise Problem Sets 9

Nov. 06. 2026

Problem 1. Compute the indefinite integral $\int \sin^6 x\,dx$ by the following steps:

1. Write $\sin^6 x = \sin^3 x \cdot \sin^3 x$, and use the triple and double angle formula, as well as the product to sum formula, to show that
$$\sin^6 x = -\frac{1}{32}\cos 6x + \frac{3}{16}\cos 4x - \frac{15}{32}\cos 2x + \frac{5}{16}.$$
2. Find the indefinite integral $\int \sin^6 x\,dx$ using the identity above.

Problem 2. Compute the indefinite integral $\int \cos^5 x\,dx$ by the following steps:

1. Write $\cos^5 x = \cos^3 x \cdot \cos^2 x$, and use the triple and double angle formula, as well as the product to sum formula, to show that
$$\cos^5 x = \frac{1}{16}\cos 5x + \frac{5}{16}\cos 3x + \frac{5}{8}\cos x.$$
2. Find the indefinite integral $\int \cos^5 x\,dx$ using the identity above.

Problem 3. Find the indefinite integral $\int \left(\frac{\sin x - \cos x}{\sin x + \cos x}\right)^4 dx$.
Hint: Use the substitution of variable $t = \frac{\sin x - \cos x}{\sin x + \cos x}$.

Problem 4. Find the following indefinite integrals.

1. $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$.
2. $\int \frac{1}{x^3}\cos^2\frac{1}{x^2}\,dx$.
3. $\int \frac{1}{x^2}\cos^3\frac{1}{x}\,dx$.
4. $\int \frac{\sin\sqrt{x}}{\sqrt{x\cos^3\sqrt{x}}}\,dx$.

Problem 5. Find the following indefinite integrals.

1. $\int \tan^3 x\sec^2 x\,dx$, $\int \frac{\sec^2 x}{\tan^2 x}\,dx$, $\int \tan^2 x\sec^2 x\,dx$. Use your experience on these three integrals to find $\int \tan^m x\sec^2 x\,dx$ for $m \neq -1$.
2. $\int \sec^3 x\tan x\,dx$, $\int \sec^5 x\tan x\,dx$, $\int \frac{\tan x}{\sec^3 x}\,dx$. Use your experience on these three integrals to find $\int \sec^m x\tan x\,dx$ for $m \neq 0$.

Problem 6. Let y be a twice differentiable function satisfying
$$\frac{d^2y}{dx^2} = 4\sec^{-2} 2x\tan 2x, \quad y(0) = -1, y'(0) = 4.$$
Find y.

Problem 7. 1. Let $f : [0, a] \to \mathbb{R}$ be a continuous function. Find $\displaystyle\int_0^a \frac{f(x)}{f(x) + f(a-x)}\, dx$.

2. Find $\displaystyle\int_0^1 \frac{\sin x}{\sin x + \sin(1-x)}\, dx$.

3. Find $\displaystyle\int_0^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3-x}}\, dx$.

Problem 8. Let $f : [-1, 1] \to \mathbb{R}$ be a continuous function.

1. Show that
$$\int_0^{\frac{\pi}{2}} f(\sin x)\, dx = \int_0^{\frac{\pi}{2}} f(\cos x)\, dx\,.$$

2. Use the identity above to find the integrals $\displaystyle\int_0^{\frac{\pi}{2}} \cos^2 x\, dx$ and $\displaystyle\int_0^{\frac{\pi}{2}} \sin^2 x\, dx$.

Problem 9. Let a and b be positive (rational) numbers. Show that
$$\int_0^1 x^a (1-x)^b\, dx = \int_0^1 x^b (1-x)^a\, dx\,.$$