

MA3111: Mathematical Image Processing Syllabus and Introduction



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Syllabus

- **Instructor:** Prof. Suh-Yuh Yang (楊肅煜)
Office: M315, Hong-Jing Hall
Phone: 03-4227151 extension 65130
- **Office hours:** Tuesday 10:00 ~ 12:00 am or by appointment.
- **Teaching assistant:** 鐘仁宏
Email: 113221020@cc.ncu.edu.tw
- **Prerequisites:** MA1018/MA2030/MA2044, and knowledge of programming language MATLAB
- **Assignments:** will be assigned approximately every two or three weeks and *announced at ee-class*. The students are encouraged to discuss homework with other classmates. *Direct copying is absolutely not allowed.*
- **Grading policy:** *assignments 30%, midterm 30%, final 30%, and others 10%.*

Course objective

This course is concerned with the mathematical study of image processing. Its two main objectives are

- (1) *to introduce basic concepts and engineering approaches applicable to digital image processing and develop a further study foundation.*
- (2) *to provide some mathematical techniques for studying several fundamental questions in image processing, such as how to restore a degraded image and how to segment it into meaningful regions.*

No textbook but some references

- (1) [AK2006] G. Aubert and P. Kornprobst, *Mathematical Problems in Image Processing: Partial Differential Equations and the Calculus of Variations, Second Edition*, Springer Verlag, New York, 2006.
- (2) [CS2005] T. F. Chan and J. Shen, *Image Processing and Analysis: Variational, PDE, Wavelet, and Stochastic Methods*, Society for Industrial and Applied Mathematics, Philadelphia, 2005.
- (3) [TUM2019] D. Cremers, *Computer Vision I: Variational Methods, Online Resources*, Departments of Informatics & Mathematics, Technical University of Munich, Germany, 2019/2020.
<https://vision.in.tum.de/teaching/online/cvvm>
- (4) [GW2018] R. C. Gonzalez and R. E. Woods, *Digital Image Processing, Fourth Edition*, Pearson Education Limited, New York, 2018.

Important dates

- The period for adding and dropping a course: 09/02-09/22, 2026
- The period for withdrawing a course: 10/19-11/27, 2026
- Midterm: 11/03 (Tue), 2026 (9th week)
- Sports games: November 11 (Wed), 2026, *no class!*
- Final: 12/22 (Tue), 2026 (16th week)

Image processing

- Continuous and discrete (digital) images:
 - (1) A grayscale image is a real-valued function $f : \overline{\Omega} \rightarrow \mathbb{R}_0^+$. A color image is a vector-valued function $f : \overline{\Omega} \rightarrow (\mathbb{R}_0^+)^3$.
 - (2) The result of sampling and quantization is a real matrix (矩陣) for a grayscale image and a real tensor (張量) for a color image.
- Image processing is a multidisciplinary field. It overlaps with other areas such as *image analysis/understanding* (影像分析/影像了解) and *computer vision* (電腦視覺).

**image processing $\implies$ image analysis/understanding
 $\implies$ computer vision**

This course will cover the following topics

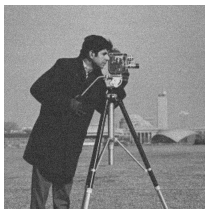
- (1) Basic concepts of digital image processing
- (2) Intensity transformations and spatial filtering
- (3) Variational methods for image denoising
- (4) Variational methods for image deblurring
- (5) Multi-focus image fusion and guided filter
- (6) Image dehazing: atmospheric scattering model and dark channel prior
- (7) Variational methods for image contrast enhancement
- (8) Image inpainting: variational methods and sparse representation
- (9) Image segmentation: Mumford-Shah and Chan-Vese models
- (10) Principal component pursuit problems

Image editing, image stitching, optical flow estimation, video foreground extraction, etc.

Image denoising



(a) original



(b) noisy



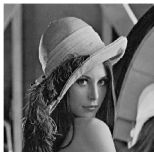
(c) ROF



(d) adaptive

Image deblurring

Original image size = 512x512



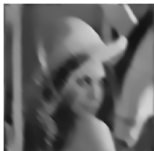
Blurry image (SNR 5.9065)



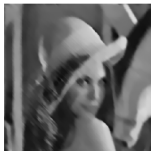
Blurry and noisy image (SNR 5.5328)



$\lambda = 10$ (SNR 7.2649)



$\lambda = 50$ (SNR 9.0242)



$\lambda = 250$ (SNR 10.9436)

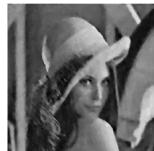


Image fusion



(e) Source1



(f) Source2



(g) LSDGF1



(h) LSDGF2

Image dehazing (defogging)



(L) hazed images; (R) dehazed images

https://www.cs.huji.ac.il/~raananf/projects/dehaze_cl/results/

Contrast enhancement



(L) low-light images; (R) contrast enhanced images

Image inpainting

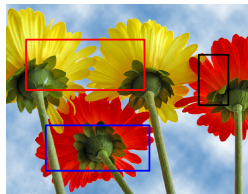


(L) original images; (M) mask images; (R) inpainted images

Image segmentation



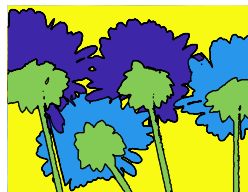
(i)



(j)



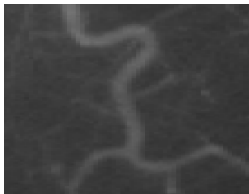
(k)



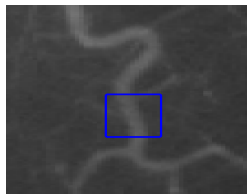
(l)

(i) input image; (j) initial contours; (k)&(l) segmented images

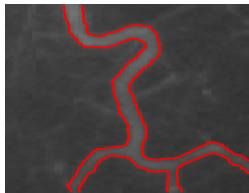
Medical image segmentation (variational method)



(m)



(n)



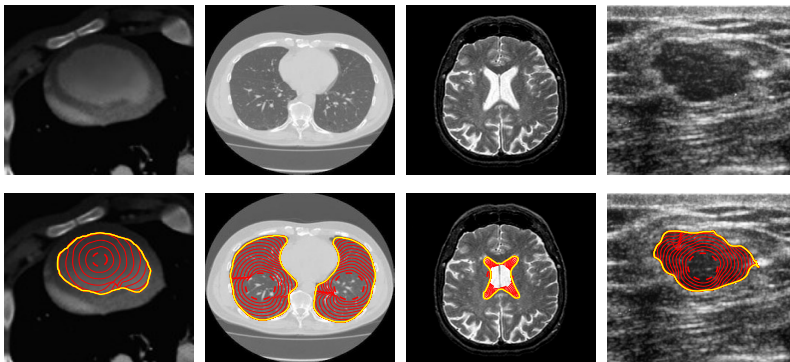
(o)



(p)

(m) input image; (n) initial contour; (o)&(p) segmented images

Medical image segmentation (snake method)



(T) Human's cardiac CT, human's lung CT, brain CT, and ultrasound
(B) Deformation processes by AeGVF

Image editing

The objective is to seamlessly clone a region selected from an image over a background image.



Panorama: image stitching



(T) 8 source images; (B) stitched image

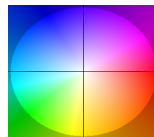
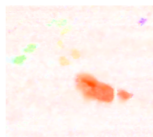
Optical flow estimation



frame 1



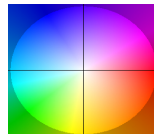
frame 2



frame 1



frame 2



Video foreground extraction

