A. 計算與證明題:

1. Prove by ϵ-δ definition: $\lim_{x\to 2}(x^3-2x+1)=5$
2. Find the limits:

   (1) $\lim_{x\to 3}\dfrac{\sqrt{x+6}-3}{x-3}$ (2) $\lim_{x\to\infty}(\sqrt{x^2+x-1}-\sqrt{x^2-1})$

   (Hint: 先有理化)
3. Use Sqeeze theorem to show: $\lim_{x\to 0} x^3\sin\dfrac{1}{x}=0$
4. Let $f(x)=\begin{cases} x^2+1, & x<1 \\ x+1, & x>1 \end{cases}$

   Find $\lim_{x\to 1^-} f(x)$, $\lim_{x\to 1^+} f(x)$ and $\lim_{x\to 1} f(x)$. If f continuous at 1 then find the value of $f(1)$.
5. Show that $x-2\sin x=0$ has a root in $(0,3)$.
6. Differentiate:

   (1) $f(x)=\dfrac{\sin x+\cos x}{x^2+1}$ (2) $y=(\cot x+\sec x)^{100}$

   (3) $g(x)=\tan\sqrt{x^2+1}$
7. Find the tangent to the graph of $y^3+3xy^2-x^3=3$ at point $(1,1)$
8. Prove: If f is continuous on $[a,b]$ and $f'(x)=0$ on (a,b) then f is a constant on $[a,b]$
9. Let $f(x)=3x^4+4x^3$. Sketch the graph of f and find intervals on which f is increasing, decreasing, concave up, concave down. Find inflection points, relative extrema and absolute extrema of f.

B. 填充題:

(1) $\lim_{x\to -\infty}\dfrac{2x-1}{\sqrt{2x^2+1}}=$ ________ (2) $\lim_{x\to 5^-}\dfrac{1}{x^2-25}=$ ______ 。

(3) If $h(x)=\dfrac{x^3-2}{x^2-x-2}$ then vertical asymptotes are ______ and slant asymptote is ______ 。

(4) Let $f(x)=\frac{x}{x^2-3x}$ Then the removable discontinuities of f are ________. The nonremovable discontinuities of f are ________.

(5) If $f(x)=x^2\sin\frac{1}{x}$ then $f'(0)=$ ______ 。

(6) True or false:

(a) $\lim_{x\to c} f(x)=L \Leftrightarrow \lim_{x\to c}|f(x)|=|L|$ True or false: ______

(b) $\lim_{x\to c} f(x)=0 \Leftrightarrow \lim_{x\to c}|f(x)|=0$ True or false: ______

(c) $f(x)>0$ on (a,b) and $c\in(a,b)\Rightarrow \lim_{x\to c} f(x)>0$ True or false: ______

(d) If f is differentiable at c and $f'(c)=0$ then f has a local extrema at c.